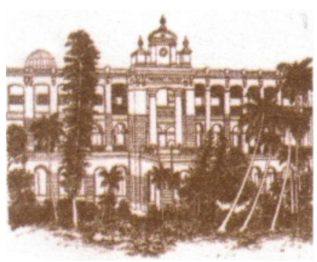
PRESIDENCY UNIVERSITY

DEPARTMENT OF MATHEMATICS

Syllabus for B.Sc. (Hons.) Mathematics
under Choice Based Credit System (CBCS)
(effective from Academic Year 2018-19)

PRESIDENCY
UNIVERSITY
KOLKATA



Department of Mathematics
(Faculty of Natural and Mathematical Sciences)
Presidency University
Hindoo College (1817-1855), Presidency College (1855-2010)
86/1, College Street, Kolkata - 700 073
West Bengal, India

Department of Mathematics
Presidency University

Course Structure for B.Sc. (Hons.) Mathematics under CBCS

Semester	Core Course (14)	Ability Enhancement Compulsory Course (AECC) (2)	Skill Enhancement Course (SEC) (2)	Discipline Specific Elective (DSE) (4)	General Elective (GE) (4)
I	**MATH 01C1** Calculus and Geometry	**AECC-1** English Language			**MATH 01GE1** Calculus I
	MATH 01C2 Algebra				
II	**MATH 02C3** Real Analysis I	**AECC-2** Environmental Science			**MATH 02GE2** Calculus II
	MATH 02C4 Groups and Rings I				
III	**MATH 03C5** Real Analysis II		**MATH 03SEC1** Computer Programming		**MATH 03GE3** Algebra
	MATH 03C6 Linear Algebra I				
	MATH 03C7 Ordinary Differential Equations				
IV	**MATH 04C8** Sequence and Series of Functions and Metric Spaces		**MATH 04SEC2** LaTeX		**MATH 04GE4** Analytical Geometry
	MATH 04C9 Multivariate Calculus				
	MATH 04C10 Partial Differential Equations				
V	**MATH 05C11** Numerical Methods			**MATH 05DSE1**	
	MATH 05C12 Groups and Rings II			**MATH 05DSE2**	
VI	**MATH 06C13** Complex Analysis and Fourier Series			**MATH 06DSE3**	
	MATH 06C14 Probability Theory			**MATH 06DSE4**	

Details of the Credit Structure of Courses:

Sl. No.	Course	Credit	Theory	Tutorial/Practical	Marks*
1	Core Course	6×14	$5 \times 13 + 4 \times 1$	$1 \times 13 + 2 \times 1$	$100 \times 14 = 1400$
2	DSE	6×4	$5 \times 4\ (5 \times 3 + 4 \times 1)$	$1 \times 4\ (1 \times 3 + 2 \times 1)$	$100 \times 4 = 400$
3	GE	6×4	5×4	1×4	$100 \times 4 = 400$
4	SEC	4×2	4×2		$100 \times 2 = 200$
5	AECC	4×2	4×2		$100 \times 2 = 200$
	Total	**148**	**125**	**23**	**2600**

* For the core course `MATH 05C11` and the DSE course `MATH 05DSE2-B`, division of marks would be 100=70(end semester exam.)+30(practical exam.) and for all the other courses the division of marks would be 100=80(end semester exam.)+20(internal assessment).

Discipline Specific Electives (DSE):

1. **Choices for MATH 05DSE1**
   (a) MATH 05DSE1-A: Linear Programming and Game Theory
   (b) MATH 05DSE1-B: Number Theory
2. **Choices for MATH 05DSE2**
   (a) MATH 05DSE2-A: Theory of Ordinary Differential Equations (ODE)
   (b) MATH 05DSE2-B: Mathematical Modelling
3. **Choices for MATH 06DSE3**
   (a) MATH 06DSE3-A: Linear Algebra II and Field Theory
   (b) MATH 06DSE3-B: Industrial Mathematics
4. **Choices for MATH 06DSE4**
   (a) MATH 06DSE4-A: Mechanics
   (b) MATH 06DSE4-B: Differential Geometry

Detailed Syllabi of the Courses

Core 1: Calculus and Geometry
Subject Code: MATH 01C1
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Calculus: Hyperbolic functions, higher order derivatives, Leibniz rule and its applications to problems of type $e^{ax+b}\sin x$, $e^{ax+b}\cos x$, $(ax+b)^n\sin x$, $(ax+b)^n\cos x$, concavity and inflection points, asymptotes, curve tracing in Cartesian coordinates, tracing in polar coordinates of standard curves, L'Hospital's rule, applications in business, economics and life sciences.

Reduction formulae, derivations and illustrations of reduction formulae of the type $\int \sin nx dx$, $\int \cos nx dx$, $\int \tan nx dx$, $\int \sec nx dx$, $\int (\log x)^n dx$, $\int \sin^n x \sin^m x dx$, volumes by slicing, disks and washers methods, volumes by cylindrical shells, parametric equations, parametrizing a curve, arc length, arc length of parametric curves.

Triple product, introduction to vector functions, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions, tangent and normal components of acceleration, modeling ballistics and planetary motion, Kepler's second law.

Geometry: Techniques for sketching parabola, ellipse and hyperbola. Properties of parabola, ellipse and hyperbola, polar equations of conics, classification of conics using the discriminant.

Equation of a plane, signed distance of a point from a plane, planes passing through the intersection of two planes, angle between two intersecting planes and their bisectors. Parallelism and perpendicularity of two planes. Equations of a line in space, rays or half lines, direction cosines of a line, angle between two lines, distance of a point from a line, condition for coplanarity of two lines, skew-lines, shortest distance. Spheres, cylindrical surfaces, cone, ellipsoid, surface of revolution.

Books Recommended:

1. G.B. Thomas and R.L. Finney, Calculus, Pearson.
2. M.J. Strauss, G.L. Bradley and K. J. Smith, Calculus, Pearson.
3. H. Anton, I. Bivens and S. Davis, Calculus, John Wiley.
4. R. Courant and F. John, Introduction to Calculus and Analysis, I & II, Springer.
5. T. Apostol, Calculus, I & II, John Wiley.
6. S.L. Loney, The Elements of Coordinate Geometry, McMillan.
7. J.T. Bell, Elementary Treatise on Coordinate Geometry of Three Dimensions, McMillan.

Core 2: Algebra
Subject Code: MATH 01C2
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Quick review of algebra of complex numbers, modulus and amplitude (principal and general values) of a complex number, polar representation, De-Moivre's Theorem and its applications: nth roots of unity.

Polynomials with real coefficients and their graphical representation. Relationship between roots and coefficients: Descarte's rule of signs, symmetric functions of the roots, transformation of equations. Solutions of the cubic and bi-quadratic equations. Statement of the fundamental theorem of Algebra. Inequality: The inequality involving AM$\geq$GM$\geq$HM, Cauchy-Schwartz inequality.

Equivalence relations and partitions, partial order, statement of Zorn's lemma. Mappings and functions, injective, surjective and bijective mappings, composition of mappings, invertible mappings. Cardinality of a set, countable and uncountable sets, bijection from the unit interval to unit square using Schroeder-Bernstein theorem, well ordering principle. Divisibility and Euclidean algorithm, congruence relation between integers. Principle of mathematical induction. Statement of the fundamental theorem of arithmetic.

Elementary row operations: row reductions, elementary matrices, echelon forms of a matrix, rank of a matrix, characterization of invertible matrices using rank. Solution of systems of linear equations $A\mathbf{x} = \mathbf{b}$: Gaussian elimination method and matrix inversion method.

Elements of $\mathbb{R}^n$ as vectors, linear combination and span of vectors in $\mathbb{R}^n$, linear independence and basis, vector subspaces of $\mathbb{R}^n$, dimension of subspaces of $\mathbb{R}^n$. Linear transformations on $\mathbb{R}^n$ as structure preserving maps, invertible linear transformations, matrix of a linear transformation, change of basis matrix. Adjoint, determinant and inverse of a matrix.

Definitions and examples: (i) groups, subgroups, cosets, normal subgroups, homo-morphisms. (ii) rings, subrings, integral domains, fields, characteristic of a ring, ideals, ring homomorphisms.

Books Recommended:

1. Bernard and Child, Higher Algebra, Macmillan.
2. S. K. Mapa, Classical Algebra, Levant.
3. T. Andreescu and D. Andrica, Complex Numbers from A to Z, Birkhauser.
4. D. C. Lay, Linear Algebra and its Applications, Pearson.
5. C. Curtis, Linear Algebra, Springer.
6. J. B. Fraleigh, A First Course in Abstract Algebra, Pearson.
7. Ghosh, Mukhopadhyay and Sen, Topics in Abstract Algebra, University Press.

Core 3: **Real Analysis I**
Subject Code: **MATH 02C3**
Credits: **6** (Theory-5, Tutorial-1)
Contact Hours per Week: **6** (5 Theory lectures + 1 Tutorial)

Review of Algebraic and Order Properties of $\mathbb{R}$, δ-neighbourhood of a point in $\mathbb{R}$, countability of $\mathbb{Q}$ and uncountability of $\mathbb{R}$. Bounded above sets, Bounded below sets, Bounded Sets, Unbounded sets, Supremum and Infimum, The Completeness Property of $\mathbb{R}$, The Archimedean Property, Density of Rational (and Irrational) numbers in $\mathbb{R}$, Intervals. Limit points of a set, Isolated points, Illustrations of Bolzano-Weierstrass theorem for sets.

Sequences, Bounded sequence, Convergent sequence, Limit of a sequence, Limit superior and Limit inferior, Limit Theorems, Monotone Sequences, Monotone Convergence Theorem. Subsequences, Divergence Criteria, Monotone Subsequence Theorem (statement only), Bolzano-Weierstrass Theorem for Sequences. Cauchy sequence, Cauchys Convergence Criterion. Construction of $\mathbb{R}$ from $\mathbb{Q}$ by Dedekind's cut or by equivalent Cauchy sequences.

Infinite series, convergence and divergence of infinite series, Cauchy Criterion, Tests for convergence: Comparison test, Limit Comparison test, Ratio Test, Cauchy's root test, integral test, Alternating series, Leibniz test, Absolute and Conditional convergence, Cauchy product, Rearrangements of terms, Riemann's theorem on rearrangement of series (statement only).

Limits of functions (ϵ-δ approach), sequential criterion for limits, divergence criteria. Limit theorems, one sided limits. Infinite limits and limits at infinity.

Continuous functions, sequential criterion for continuity and discontinuity. Algebra of continuous functions. Continuous functions on an interval, intermediate value theorem, location of roots theorem, preservation of intervals theorem. Uniform continuity, non-uniform continuity criteria, uniform continuity theorem.

Books Recommended:

1. R.G. Bartle and D. R. Sherbert, Introduction to Real Analysis, J. Wiley.

2. Terence Tao, Analysis I, HBA.

3. Walter Rudin, Principles of Mathematical Analysis, McGraw-Hill.

4. Tom Apostol, Mathematical Analysis, Narosa.

5. S.K. Berberian, A First Course in Real Analysis, Springer Verlag

Core 4: Groups & Rings I
Subject Code: MATH 02C4
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Groups: Groups as symmetries, examples: S_n, D_n, $GL(n, \mathbb{R})$, $SL(n, \mathbb{R})$, $O(n, \mathbb{R})$, $SO(n, \mathbb{R})$ etc., elementary properties of groups, abelian groups.

Subgroups and cosets: examples of subgroups including centralizer, normalizer and center of a group, product subgroups; cosets and Lagrange's theorem with applications. Cyclic groups and its properties, classification of subgroups of cyclic groups. Normal subgroups: properties and examples, conjugacy classes of elements, properties of homomorphisms and kernels, quotient groups and their examples. Presentation of a group in terms of generators and relations.

Properties of S_n, cycle notation for permutations, even and odd permutations, cycle decompositions of permutations in S_n, alternating group A_n, Cayley's theorem. Isomorphism theorems: proofs and applications, isomorphism classes of finite groups of lower order. Cauchy's theorem for finite abelian groups, statements of Cauchy's and Sylow's theorem and applications.

Rings: Examples and basic properties of rings, subrings and integral domains. Ideals, algebra of ideals, quotient rings, chinese remainder theorem. Prime and maximal ideals, quotient of rings by prime and maximal ideals, ring homomorphisms and their properties, isomorphism theorems, field of fractions.

Books Recommended:

1. Herstein, Topics in Algebra, John Wiley.

2. Fraleigh, A First Course in Abstract Algebra, Pearson.

3. M. Artin, Algebra, Pearson.

4. Bhattacharya, Jain and Nagpaul, Basic Abstract Algebra, Cambridge Univ. Press.

5. Gallian, Contemporary Abstract Algebra, Narosa.

6. Rotman, An Introduction to the Theory of Groups, Springer.

7. Dummit and Foote, Abstract Algebra, John Wiley.

8. Hungerford, Algebra, Springer.

Core 5: Real Analysis II
Subject Code: MATH 03C5
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Differentiability of a function at a point and in an interval, Carathéodory's theorem, Algebra of differentiable functions, Chain rule, Relative extrema, interior extremum theorem. Rolle's theorem, Mean value theorem, intermediate value property of derivatives - Darboux's theorem. Cauchy's mean value theorem. Applications of mean value theorems to inequalities and approximation of polynomials. Proof of L'Hôpital's rule.

Taylor's theorem with Lagrange's form of remainder and Cauchy's form of remainder, application of Taylor's theorem to convex functions. Taylor's series and Maclaurin's series expansions of exponential, trigonometric and other functions.

Riemann integration. upper and lower sums, Riemann's conditions of integrability. Riemann sum and definition of Riemann integral through Riemann sums. Equivalence of the two definitions. Riemann integrability of monotone and continuous and piecewise continuous functions. Properties of the Riemann integral. Intermediate Value theorem for integrals. Fundamental theorems of Calculus and its consequences. Functions of bounded variation and their properties.

Improper integrals; Proof of integral test for series, Convergence of Beta and Gamma functions, Bohr-Mollerup theorem and its consequences.

Books Recommended:

1. R.G. Bartle and D.R. Sherbert, Introduction to Real Analysis, John Wiley.
2. Terence Tao, Analysis I & II, HBA.
3. Tom Apostol, Mathematical Analysis, Narosa.
4. Tom Apolstol, Calculus I, Wiley.
5. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill.
6. C. G. Denlinger, Elements of Real Analysis, Jones & Bartlett.
7. K.A. Ross, Elementary Analysis, The Theory of Calculus, Springer (UTM).
8. Royden, Real Analysis, Pearson.

Core 6: Linear Algebra I
Subject Code: MATH 03C6
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Vector spaces, linear combination of vectors, linear span, linear independence, basis and dimension. Subspaces, algebra of subspaces, dimension of subspaces, quotient spaces.

Linear transformations, matrix representation of a linear transformation, null-space and range, rank and nullity of a linear transform, Sylvester's (rank-nullity) theorem and application. Algebra of linear transformations, isomorphisms and isomorphism theorems, change of coordinate matrix.

Row space and column space of a matrix, row rank, column rank and rank of a matrix, equality of these ranks, rank of product of two matrices, rank factorisation.

Linear functionals, dual spaces, dual basis, double dual, transpose of a linear transform and its matrix in the dual basis, annihilators.

Characteristic polynomial; eigen values, eigen vectors and eigen space of a linear transform, diagonalizability of a matrix, invariant subspaces and Cayley-Hamilton theorem. Minimal polynomial for a linear operator, diagonalizability in connection with minimal polynomial, canonical forms.

Inner products and norms; inner products spaces, Cauchy-Schwarz inequality, orthogonal and orthonormal basis, orthogonal projections, orthogonal complements, Gram-Schmidt orthogonalisation process, Bessel's inequality.

Definitions of real symmetric, orthogonal, Hermitian, normal, unitary matrices; spectral theorems for real symmetric matrices.

Books Recommended:

1. K. Hoffman, R. A. Kunze, Linear Algebra, PHI.
2. S. Lang, Introduction to Linear Algebra, Springer.
3. Gilbert Strang, Linear Algebra and its Applications, Academic.
4. S. Kumaresan, Linear Algebra: A Geometric Approach, PHI.
5. P.R. Halmos, Finite Dimensional Vector Spaces, Springer.
6. A. R. Rao and P. Bhimasankaram, Linear Algebra, HBA.
7. Friedberg, Insel and Spence, Linear Algebra, Pearson.
8. C. Curtis, Linear Algebra: An Introductory Approach, Springer (UTM).

Core 7: Ordinary Differential Equations
Subject Code: MATH 03C7
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Formation of ordinary differential equations, geometric interpretation, general solution, particular solution.

First order equations: Lipschitz condition, linear first-order equations, exact equations and integrating factors, separable equations, linear and Bernoulli forms. Higher degree equations: general solution and singular solution, Clairut's form, singular solution.

Higher order equations: second order equations, general theory and solution of a homogeneous equation, Wronskian (properties and applications), general solution of a non-homogeneous equation, Euler-Cauchy forms, method of undermined coefficients, normal form, variation of parameters, use of f(D) operator, solution of both homogeneous and inhomogeneous higher order equations (order greater than two).

Strum-Liouville problem, eigenvalues and eigenfunctions.

Systems of linear differential equations: basic theory, normal form, homogeneous linear systems with constant coefficients.

Power series solution: solution about regular singular point, applications: hypergeometric and Legendre differential equations, properties of both functions.

Books Recommended:

1. Murray: Introductory Course on Differential Equations.

2. S. L. Ross: Differential Equations.

3. H.T.H. Piaggio: Differential Equations.

4. G.F. Simmons: Differential Equation with Applications and Historical Notes.

5. A.C. King, J. Billingham and S.R. Otto: Differential Equations: Linear, Nonlinear, Ordinary, Partial.

6. Edwards and Penny: Differential equations and boundary value problems: Computing and Modelling.

Core 8: Sequence & Series of Functions and Metric Spaces
Subject Code: MATH 04C8
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Sequence & series of functions: Point-wise and uniform convergence of sequence of functions. Theorems on convergence of a sequence of functions and continuity, differentiability and integrability of the corresponding limit function. Series of functions; Theorems on the continuity and differentiability of the sum function of a series of functions. Cauchy criterion for uniform convergence and Weierstrass' M-Test, construction of nowhere differentiable continuous maps on $\mathbb{R}$.

Power series, radius of convergence, Cauchy-Hadamard Theorem, Differentiation and integration of power series; Abel's Theorem; Weierstrass' Approximation Theorem.

Metric spaces: definition and examples. $\mathbb{R}^n$ as a metric space. Elementary topological notions for metric spaces: Open and closed balls, Neighbourhood of a point, Interior of a set, Limit point of a set, Open and Closed sets in a metric space, Dense subsets of a metric space. Separable spaces. Discrete metric space.

Sequences in metric spaces and Cauchy sequences, Completeness, Completion of a general metric space. The Baire Category Theorem.

Compact metric spaces, Compact subsets of $\mathbb{R}^n$, The Bolzano-Weierstrass theorem, Cantor's theorem, Supremum and Infimum on compact sets, Total boundedness, equivalence of sequential compactness with compactness for metric spaces. Connectedness, Connected subsets of $\mathbb{R}$. Continuity and connectedness, Continuous and uniformly continuous functions on a metric space. Sequential criterion of continuity. Homeomorphisms.

Contraction mappings, Banach contraction principle. C(X) as a metric space. Arzelá-Ascoli Theorem. Stone-Weierstrass Theorem (statement only).

Books Recommended:

1. Terence Tao, Analysis II, HBA (TRIM Series).
2. Tom Apostol, Mathematical Analysis, Narosa.
3. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill.
4. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill.
5. S. Kumaresan, Topology of Metric Spaces, Narosa.
6. Irving Kaplansky, Set Theory and Metric Spaces, AMS Chelsea Publishing.
7. S. Shirali and H. L. Vasudeva, Metric Spaces, Springer.
8. J. Munkres, Topology, Pearson.

Core 9: Multivariate Calculus
Subject Code: MATH 04C9
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Functions of several variables, limit and continuity, partial derivatives, differentiability and total derivatives as matrices, sufficient condition for differentiability, chain rule; gradient vector, directional derivatives, the Jacobian theorem; inverse and implicit function theorems; higher derivatives and Taylor's theorem.

Maxima and minima, constrained optimisation problems, Lagrange's multipliers.

Tangent spaces, definition of a vector field, divergence and curl of a vector field, identities involving gradient, curl and divergence; maximality and normality properties of the gradient vector field.

Double integrals, double integrals: (i) over Rectangular region, (ii) over non-rectangular regions, (iii) in polar co-ordinates; triple integrals, triple integrals over: (i) a parallelepiped, (ii) solid regions; computing volume by triple integrals, in cylindrical and spherical co-ordinates; change of variables in double integrals and triple integrals.

Line integrals, applications of line integrals: mass and work, fundamental theorem for line integrals, conservative vector fields, independence of path. Green's theorem, Surface integrals, Stoke's theorem, The Divergence theorem.

Books Recommended:

1. Terence Tao, Analysis II, HBA (TRIM Series).
2. Tom Apostol, Mathematical Analysis, Narosa.
3. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill.
4. Tom Apostol, Calculus II, John Wiley.
5. M. Spivak, Calculus on Manifold.
6. E. Marsden, A.J. Tromba and A. Weinstein, Basic Multivariable Calculus, Springer.
7. J. Stewart, Multivariable Calculus, Concepts and Contexts; Thomson.

Core 10: Partial Differential Equations
Subject Code: MATH 04C10
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Partial differential equations: basic concepts and definitions, mathematical problems. first- order equations: classification, construction and geometrical interpretation; method of characteristics for obtaining general solution of quasi linear equations, canonical forms of first-order linear equations. method of separation of variables for solving first order partial differential equations.

Derivation of heat equation, wave equation and Laplace equation; classification of second order linear equations as hyperbolic, parabolic or elliptic; reduction of second order linear equations to canonical forms.

The Cauchy problem, the Cauchy-Kovalevskaya theorem, Cauchy problem of an infinite string, initial boundary value problems, semi-infinite string with a fixed end, semi-infinite string with a free end, equations with non-homogeneous boundary conditions, non-homogeneous wave equation, method of separation of variables, solving the vibrating string problem, solving the heat conduction problem.

Laplace transforms: introduction and properties with derivations; existence, simple problems, inverse Laplace transform, convolution, solving ODEs and PDEs using Laplace transform.

Books Recommended:

1. I. N. Sneddon: Elements of Partial Differential equations.

2. E. T. Copson: Partial Differential Equations.

3. T. Amarnath, An elementary course in partial differential equations, Narosa.

4. T. Myint-U and L. Debnath, Linear Partial Differential Equations for Scientists and Engineers, Springer.

Core 11: Numerical Methods
Subject Code: MATH 05C11
Credits: 6 (Theory-4, Practical-2)
Contact Hours per Week: 8 (4 Theory lectures + 4 Practical classes)

Errors (Absolute, relative, round off, truncation).

Solution of transcendental and non linear equations: bisection method, secant and Regula-Falsi methods, iterative methods, Newton's methods, convergence and errors of these methods.

Interpolation: Lagrange's and Newton's divided difference methods, Newton's forward and backward difference methods, Gregory's forward and backward difference interpolation; error bounds of these methods.

Solution of a system of linear algebraic equations: Gaussian elimination method, Gauss-Jordan, Gauss-Jacobi and Gauss-Siedel methods and their convergence analysis.

Numerical integration: trapezoidal rule, Simpson's rule, composite trapezoidal and Simpson's rule, Bolle's rule, midpoint rule, Simpson's 3/8-th rule, error analysis of these methods.

Ordinary differential equations: modified Euler's method and Runge-Kutta method of second and fourth orders.

List of Practicals (Using any software):

1. Root finding using bisection, Newton-Raphson, secant and Regula-Falsi method.
2. LU decomposition.
3. Gauss-Jacobi method.
4. Gauss-Siedel method.
5. Interpolation using Lagrange's and Newton's divided differences.
6. Integration using Simpson's Rule.
7. Differentiation using Runge-Kutta method.

Books Recommended

1. K. Atkinson: Introduction to Numerical Analysis.
2. Sastry: Introductory Methods of Numerical Analysis.
3. W. Press, S. Teukolsky, W. Vettering, B. Flannery: Numerical Recipes in C.
4. U. Ascher and C. Greif: A First Course in Numerical Methods, PHI.
5. J. Mathews and K. Fink: Numerical Methods using Matlab, PHI.
6. Jain, Iyengar, Jain: Numerical Methods for Scientific & Engineering Computation, New age.

Core 12: Groups & Rings II
Subject Code: MATH 05C12
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Groups: Group actions, stabilisers and kernels, orbit-stabiliser theorem and applications, permutation representation associated to a group action, Cayleys theorem via group action, groups acting on themselves by conjugation, class equation and consequences, conjugacy in S_n, p-groups, proof of Cauchy's theorem and Sylows theorems and consequences.

Automorphisms of a group, inner automorphisms, group of automorphisms and their computations (in particular for finite and infinite cyclic groups), characteristic subgroups, commutator subgroup and its properties.

Direct product of groups, properties of external direct products, $\mathbb{Z}_n$ as external direct product, internal direct products, fundamental theorem of finite abelian groups, fundamental theorem of finitely generated Abelian groups (statement only) and its applications.

Rings: Polynomial rings over commutative rings, division algorithm and consequences, factorisation of polynomials, reducibility tests, irreducibility tests, Eisenstein's criterion. Principal Ideal Domains (PID), unique factorisation in $\mathbb{Z}[x]$, divisibility in integral domains, irreducibles and primes, Unique Factorisation Domains (UFD), Euclidean Domains (ED).

Books Recommended:

1. Gallian, Contemporary Abstract Algebra, Narosa.
2. Fraleigh, A First Course in Abstract Algebra, Pearson.
3. M. Artin, Abstract Algebra, Pearson.
4. Hungerford, Algebra, Springer.
5. Dummit and Foote, Abstract Algebra, John Wiley.
6. Bhattacharya, Jain and Nagpaul, Basic Abstract Algebra, CUP.
7. Rotman, An Introduction to the Theory of Groups, Springer.
8. Musili, Rings and Modules, Narosa.

Core 13: Complex Analysis and Fourier Series
Subject Code: MATH 06C13
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Complex Analysis: Complex numbers, field structure of complex numbers, geometric interpretation, stereographic projection.

Functions of complex variable, mappings. Derivatives, Holomorphic functions, differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability.

Analytic functions and power series, analytic functions are holomorphic, absolute and uniform convergence of power series, examples of analytic functions, exponential function, trigonometric function, complex logarithm.

Rectifiable paths, Riemann-Steieltjes integral of a function over an interval, Definition of complex integration of functions over a rectifiable path. Contours, Contour integrals and its examples, upper bounds for moduli of contour integrals. Cauchy-Goursat theorem, Cauchy's integral formula, equivalence of analyticity and holomorphicity.

Liouville's theorem and the fundamental theorem of algebra. Zeros of analytic functions and identity principle. Morera's theorem. Convergence of sequences and series of analytic functions.

Poles, removable and essential singularity, Riemann's theorem on removable singularities, residue formula, Casorati-Weierstrass theorem, meromorphic functions. The argument principle, The open mapping property of holomorphic functions, maximum modulus principle, Schwarz lemma, conformality, Möbius transformations and the cross ratio, winding number, Laurent series.

Fourier series: Complex valued periodic functions on $\mathbb{R}$, inner products on periodic functions, trigonometric polynomials, Fourier series and coefficients, periodic convolutions, Weierstrass' approximation theorem for trigonometric polynomials, Fourier's theorem on mean square convergence, Bessel's inequality, Riemann-Lebesgue lemma, Parseval's identity, Dirichlet's theorem on convergence of Fourier series (proof can be done if time permits).

Books Recommended:

1. Brown and Churchill, Complex Variables and Applications, McGraw-Hill.
2. J.B. Conway, Functions of One Complex Variable, Springer.
3. Stein and Shakarchi, Complex Analysis, Princeton Univ. Press.
4. Gamelin: Complex Analysis, Springer.
5. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill.
6. T. Tao, Analysis II, HBA.
7. T. Apostol, Mathematical Analysis, Narosa.

Core 14: Probability Theory
Subject Code: MATH 06C14
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Classical theory and its limitations: random experiment and events, event space; classical definition of probability and its drawback, statistical regularity, frequentist probability.

Probability axioms: basics from measure theory (sets, collections of sets, generators, the monotone class theorem, probability measure, probability space, the Borel sets on $\mathbb{R}$), continuity theorem in probability, exclusion-inclusion formula, conditional probability and Bayes' rule, Boole's inequality, independence of events, Bernoulii trials and binomial law, Poisson trials, probability on finite sample spaces, geometric probability.

Random variables and their probability distributions: random variables, probability distribution of a random variable, discrete and continuous random variable, some discrete and continuous distributions on $\mathbb{R}$: Bernoulli, binomial and Poisson; uniform, normal, Gamma, Cauchy and χ^2-distributions; functions of a random variable and their probability distributions.

Characteristics and generating functions: expectation, moments, measures of central tendency, measures of dispersion, measures of skewness and Curtois, Markov, Chebycheff's inequality, probability generating function, moment generating function, characteristic function.

Probability distributions on $\mathbb{R}^n$: random vectors, probability distribution of a random vector, functions of random vectors and their probability distributions, moments, generating functions, correlation coefficient, conditional expectation, the principle of least squares, regression.

Convergence and limit theorems: sequence of random variables, convergence in distribution, convergence in probability, almost sure convergence, convergence in rth mean, weak and strong law of large numbers, Borel-Cantelli lemma, limiting characteristic functions, central limit theorem.

Introduction to stochastic processes, discrete time Markov chains, random Walk, continuous time Markov processes, Poisson process (if time permits).

Books Recommended:

1. W. Feller, An introduction to probability theory and its applications I, J. Wiley.
2. Stirzaker and Grimmett: Probability and Random Processes.
3. Rohatgi and Saleh: An introduction to probability and statistics, John Wiley.
4. Durett: Probability Theory & Examples.
5. Allan Gut: Probability-A Graduate Course, Springer.

Discipline Specific Elective Courses:

DSE 1: A. Linear Programming and Game Theory
Subject Code: MATH 05DSE1-A
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Linear Programming: General form of linear programming problems: basic formulation and geometric interpretation, standard and canonical forms.

Basic solutions, examples, feasible solutions, degenerate solutions, reduction of a feasible solution to a basic feasible solution; convex set of feasible solutions of a system of linear equations and linear in-equations; extreme points, extreme directions, and boundary points of a convex set, describing convex polyhedral sets in terms of their extreme points and extreme directions: Caratheodory's representation theorem; correspondence between basic feasible solution of a system of linear equations and extreme point of the corresponding convex set of feasible solutions.

Theory of simplex method, optimality and unboundedness, the simplex algorithm, simplex method in tableau format, complexity of the simplex method; artificial variables: two-phase method, Big-M method and their comparison; polynomial time algorithms: ellipsoidal and Karmarkar's methods.

Duality, formulation of the dual problem, primaldual relationships, economic interpretation of the dual.

Transportation problem: mathematical formulation, north-west-corner method, least cost method, and Vogel approximation method for determination of starting basic solution, algorithm for solving transportation problem. Assignment problem: mathematical formulation, Hungarian method for solving assignment problem.

Network and graph problems: minimal spanning trees, shortest path, flows in networks, perfect matching problem; Gale-Shapley algorithm for stable marriage.

Game theory: formulation of two person zero sum games, solving two person zero sum games, games with mixed strategies, graphical solution procedure, linear programming solution of games.

Books Recommended:

1. Hadley: Linear Programming, Narosa.

2. H. Karloff: Linear Programming, Modern Birkhuser Classic.

3. David Luenberger: Linear and nonlinear programming.

4. M. Osborne and A. Rubinstein: A Course in Game Theory.

5. R. Myerson: Game Theory.

6. S.R. Chakravarty, M. Mitra and P. Sarkar: A Course in Cooperative Game Theory.

DSE 1: B. Discrete Mathematics and Number Theory
Subject Code: MATH 05DSE1-B
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Combinatorics: Basic counting principles: multinomial theorem, principle of inclusion exclusion; recurrence relations and classification.

Graph Theory: Graphs and digraphs, complement, isomorphism, connectedness and reachability, adjacency matrix. Eulerian and Hamiltonian paths and circuits in graphs. Trees, rooted trees and binary trees; planar graphs, Euler's formula, statement of Kuratowski's theorem; independence number and clique number, chromatic number, statement of Havel-Hakimi Theorem and Four-color theorem.

Number Theory: Divisibility, primes and unique factorisation; GCD and Euclidean algorithm and its extension for computing multiplicative inverses; Arithmetic functions or number theoretic functions: sum and number of divisors, (totally) multiplicative functions, the greatest integer function, Euler's phi-function, Mobius function; definition and properties of the Dirichlet product; some properties of the Euler's phi-function, statement of the prime number theorem.

Linear Diophantine equations, congruences and complete residue systems; quadratic residues, quadratic reciprocity and the law of quadratic reciprocity, Euler's criterion, Legendre symbol and Jacobi symbol, Euler-Fermat theorem, Wilson's theorem, Chinese remainder theorem.

Applications: Public-key encryption, Miller-Rabin primality testing algorithm, idea of hardness of factoring and discrete logarithm problem; basics of Diffie-Hellman key agreement and RSA encryption and decryption.

Books Recommended:

1. Fred Roberts: Applied Combinatorics.
2. T. Andreescu and Z. Feng: A Path to Combinatorics for Undergraduates: Counting Strategies, Birkhauser.
3. D. B. West: Introduction to Graph Theory, PHI.
4. F. Harary: Graph Theory, Narosa.
5. D. M. Burton: Elementary Number Theory, TMH.
6. G. A. Jones and J. M. Jones: Elementary Number Theory, Springer.
7. N. Koblitz: A course in number theory and cryptography, Springer.
8. Niven, Zuckerman and Montgomery: An Introduction to the Theory of Numbers, John Wiley.

DSE 2: A. Theory of ODEs
Subject Code: MATH 05DSE2-A
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Fundamental theorem for existence and uniqueness, Gronwall's inequality, Dependence on initial conditions and parameters, maximal interval of existence, Global existence of solutions, vector fields and flows, Topological conjugacy and equivalence.

Linear flows on $\mathbb{R}^n$, The matrix exponential, linear first order autonomous systems, Jordan canonical forms, invariant subspaces, stability theory, classification of linear flows, fundamental matrix solution, non-homogeneous linear systems, periodic linear systems and Floquet theory.

α & ω Limit sets of an orbit, attractors, periodic orbits and limit cycles.

Local structure of critical points (the local stable manifold theorem, the Hartman-Grobman theorem, the center manifold theorem), the normal form theory, Lyapunov function, local structure of periodic orbits (Poincaré map and Floquet theory), the Poincaré-Benedixson theorem. Benedixson's criterion, Liénard systems.

Books Recommended:

1. C. Chicone: Ordinary differential Equations with applications.
2. L.D. Perko: Differential Equations and Dynamical Systems.
3. E. A. Coddington and N. Levinson: Theory of Ordinary Differential Equations.

DSE 2: B. Mathematical Modelling
Subject Code: MATH 05DSE2-B
Credits: 6 (Theory-4, Tutorial-2)
Contact Hours per Week: 8 (4 Theory lectures + 4 Practical classes)

Power series solution of a differential equation about an ordinary point, solution about a regular singular point, Bessel's equation and Legendre's equation, Laplace transform and inverse transform, application to initial value problem up to second order.

Monte-Carlo simulation modeling: simulating deterministic behavior (area under a curve, volume under a surface), generating random numbers: middle square method, linear congruence, queuing models: harbor system, morning rush hour, overview of optimization modeling, linear programming model: geometric solution, algebraic solution, simplex method, sensitivity analysis.

List of Practicals (using any software)

1. Plotting of Legendre polynomial for $n = 1$ to 5 in the interval [0,1]. Verifying graphically that all the roots of $P_n(x)$ lie in the interval [0,1].

2. Automatic computation of coefficients in the series solution near ordinary points.

3. Plotting of the Bessels function of first kind of order 0 to 3.

4. Automating the Frobenius Series Method.

5. Random number generation and then use it for one of the following (a) Simulate area under a curve (b) Simulate volume under a surface.

6. Programming of either one of the queuing model (a) Single server queue (e.g. Harbor system) (b) Multiple server queue (e.g. Rush hour).

7. Programming of the Simplex method.

Books Recommended

1. T. Myint-U and L. Debnath, Linear Partial Differential Equation for Scientists and Engineers, Springer.

2. F. R. Giordano, M. D. Weir and W. P. Fox, A First Course in Mathematical Modeling, Thomson Learning.

DSE 3: A. Linear Algebra II and Field Theory
Subject Code: MATH 06DSE3-A
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Linear Algebra: Quadratic forms, positive and negative definite matrices; extrema of positive definite quadratic forms. Canonical forms: rational form and Jordan form of a matrix.

Formulae of determinant and inverse of a partitioned matrix, idempotent matrices, left inverse and right inverse of full-rank rectangular matrices, generalized inverse.

Proof of spectral theorem for complex Hermitian and real symmetric matrices, singular value decomposition, polar decomposition, simultaneous diagonalization of commuting Hermitian/real symmetric matrices.

Field theory: Examples: 1) field of fractions of an integral domain, 2) field of rational polynomials. 3) field of Meromorphic functions.

Field extensions, finite and algebraic extensions, algebraic closure of a field, splitting fields; normal extensions, separable extensions, inseparable and purely inseparable extensions, simple extensions; solvability by radicals, radical extensions, ruler and compass constructions.

Finite fields: structure of finite fields, existence and uniqueness theorems; primitive elements, minimal polynomials of elements, irreducible and primitive polynomials.

Books Recommended:

1. Hoffman and Kunze, Linear Algebra, PHI.
2. Gilbert Strang, Linear Algebra and its Applications, Academic.
3. Halmos, Finite Dimensional Vector Spaces, Springer.
4. Friedberg, Insel and Spence, Linear Algebra, Pearson.
5. Dummit and Foote, Abstract Algebra, John Wiley.
6. Hungerford, Algebra, Springer.
7. Morandi, Field and Galois Theory, Springer.
8. J. Howie, Fields and Galois Theory, Springer (UMS).

DSE 3: B. Industrial Mathematics
Subject Code: MATH 06DSE3-B
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Medical imaging and inverse problems: the content is based on mathematics of X-ray and CT-scan based on the knowledge of calculus, elementary differential equations, complex numbers and matrices.

Introduction to inverse problems: “Why should we teach Inverse Problems?”
Illustration of inverse problems through problems taught in pre-calculus, calculus, matrices and differential equations, geological anomalies in earth’s interior from measurements at its surface (inverse problems for natural disaster) and (T)omography.

X-ray: introduction, X-ray behavior and Beer’s Law (the fundament question of image construction), lines in the place.

Radon Transform: definition and examples, linearity, phantom (Shepp-Logan phantom: mathematical phantoms).

Back Projection: definition, properties and examples.

CT Scan: revision of properties of Fourier and inverse Fourier transforms and applications of their properties in image reconstruction; algorithms of CT-scan machine, algebraic reconstruction techniques (abbreviated as ART) with application to CT-scan.

Books Recommended:

1. T. G. Feeman, The Mathematics of Medical Imaging, A Beginners Guide, Springer.

2. C.W. Groetsch, Inverse Problems, Activities for Undergraduates, MAA.

3. A. Kirsch, An Introduction to the Mathematical Theory of Inverse Problems.

DSE 4: A. Mechanics
Subject Code: MATH 06DSE4-A
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Statics: coplanar forces, astatic equilibrium, friction, principle of virtual work, stable and unstable equilibrium, centre of gravity for different bodies, general conditions of equilibrium, central forces and forces in three dimensions.

Particle dynamics: Newton's equation of motion of a particle, simple illustrations: simple harmonic motion, particle in a central field; central orbits and Kepler's laws, constrained motion, oscillatory motion, motion of simple pendulum.

Rigid-body dynamics: moments and products of inertia, D'Alembert's principle of motion, compound pendulum, motion in two dimensions, conservation of linear and angular momentum, conservation of energy; derivation of Lagrange's equation for conservative holonomic system from D'Alembert's principle and from variational principle; solution of problems by Lagrange's equation.

Books Recommended:

1. S. L. Loney: Elements of Statics and Dynamics 1 and 2.
2. S. L. Loney: An Elementary treatise on Dynamics of particle and rigid bodies.
3. F. Chorlton: Textbook of Dynamics.
4. N. Rana and P. Joag: Classiscal Mechanics, McGraw-Hill.
5. H. Goldstein: Classical Mechanics, Pearson.

DSE 4: B. Differential Geometry
Subject Code: MATH 06DSE4-B
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Review of multivariate calculus: directional and total derivatives, inverse & implicit function thms.

Theory of curves: parametrized curves, regular curves and arc lengths, reparametrization of regular curves, planer curves, curvature as an invariant of a planar curve; spatial curves, torsion of a spatial curves, Serret-Frenet formulae; Isoperimetric inequality, the four vertex theorem.

Theory of surfaces: definition of smooth parametrized surfaces in $\mathbb{R}^3$; examples: (regular) level surfaces, quadric surfaces, ruled surfaces, surfaces of revolution; smooth functions on a surface, smooth curves on a surface, tangent planes (in particular of level surfaces in $\mathbb{R}^3$), derivative of a smooth map and Jacobian matrix, gradient vectors, smooth vector fields on surfaces in $\mathbb{R}^3$, tangent vector fields and integral curves, first fundamental forms and length of curves, isometries of surfaces, conformal mappings of surfaces, normal vectors and orientation of a surface.

Geometry of surfaces: Geodesics - definition and example (in particular, geodesics on a surface of revolution), second fundamental form, Gauss and Weingarten maps and their properties, normal and geodesic curvatures, parallel transport and covariant derivative, Christoffel symbols, Gaussian and mean curvature, principal curvatures of a surface, flat surfaces, surfaces of constant mean curvature, Gaussian curvature of compact surfaces (in $\mathbb{R}^3$). Basic properties of geodesics and geodesic coordinates. The Gauss' equation and Codazzi-Mainardi equations, Gauss' Theorema Egregium, surfaces of constant Gaussian curvature.

Differential forms in $\mathbb{R}^3$, exterior product and exterior derivative of forms, closed and exact forms, Poincaré lemma. Forms on surfaces, integration on surfaces, Stoke's theorem, Gauss-Bonnet theorem for compact surfaces.

Books Recommended

1. Thorpe: Elementary Topics in Differential Geometry, Springer.
2. do Carmo: Differential Geometry of Curves and Surfaces, Dover.
3. Pressley: Elementary Differential Geometry, Springer (UMS).
4. do Carmo: Differential Forms and Applications, Springer (Universitext).

Skill Enhancement Courses:

SEC 1: Computer Programming
Subject Code: MATH 03SEC1
Credits: 4 (Theory-2, Tutorial-2)
Contact Hours per Week: 4 (2 Theory lectures + 2 Tutorials)

Introduction to programming in the C language: arrays, pointers, functions, recursive pro- gramming and linked lists; notion of algorithms and their complexity, order notation; lists, stacks, queues and trees; searching and sorting algorithms; object-oriented programming and introduction to C++

Books Recommended

1. B. Kernighan & D. Ritchie, The C programming Language.

2. E. Horowitz and S. Sahani, Fundamentals of Data Structure.

SEC 2: Latex
Subject Code: MATH 04SEC2
Credits: 4 (Theory-2, Tutorial-2)
Contact Hours per Week: 4 (2 Theory lectures + 2 Tutorials)

Latex: Introduction to Latex; Document structure; Typesetting text, math formulas and expressions; Tables; Figures; Equations; Bibtex; Beamer presentation.

Books Recommended:

1. H. Kopka & P.W. Daly, Guide to Latex.

2. S. Kottwitz, Latex Beginner's Guide.

General Elective Courses (to be offered to the students of other departments):

GE 1: Calculus - I
Subject Code: MATH 01GE1
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Real Numbers: Axiomatic defnition. Intuitive idea of completeness.

Real-valued functions defined on an interval : Limit of a function (Cauchy's defnition). Algebra of limits. Continuity of a function at a point and in an interval. Acquaintance with the important properties of continuous functions on closed intervals.

Derivative its geometrical and physical interpretation. Sign of derivative Monotonic increasing and decreasing functions. Relation between continuity and differentiability.

Successive derivative (Leibnitz's Theorem and its application).

Mean Value Theorems and expansion of functions like e^x; $\sin x$; $\cos x$; $(1 + x)^n$; $\ln(1 + x)$ (with validity of regions).

Applications of Differential Calculus : Maxima and Minima, Tangents and Normals, Pedal equation of a curve. De
nition and examples of singular points (viz. Node, Cusp, Isolated point).

Indeterminate Forms : L'Hospital's Rule.

Sequence of real numbers: convergence, Cauchy criteria and other elementary properties. Series of real number, Absolute and conditional convergence of series.

Books Recommended:

1. S. Bartle, Introduction to Real Analysis.
2. T.M.Apostol, Calculus (Vol. I).
3. D. Widder, Advanced Calculus.
4. Shanti Narayan, Differential Calculus.

GE 2: Calculus - II
Subject Code: MATH 02GE2
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Integration of the form $\int \frac{dx}{a+b\cos x}$, $\int \frac{l\sin x+p\cos x}{m\sin x+n\cos x}dx$ and integration of rational functions.

2. Evaluation of definite integrals.

Integration as the limit of a sum (with equally spaced as well as unequally spaced intervals)

Reduction formulae of $\int \sin^m x \cos^n x dx$; $\int \tan^n x dx$ and $\int \frac{\sin^m x}{\cos^n x}dx$ and associated problems (m and n are non-negative integers).

Definition of Improper Integrals : Statements of (i) μ-test, (ii) Comparison test. Use of Beta and Gamma functions.

Applications: rectification, quadrature, finding c.g. of regular objects, volume and surface areas of solids formed by revolution of plane curve and areas.

Order and solution of an ordinary differential equation (ODE) in presence of arbitrary constants. Formation of ODE.

First order differential equations : (i) Variables separable. (ii) Homogeneous equations and equations reducible to homogeneous forms. (iii) Exact equations and those reducible to such equation. (iv) Euler's and Bernoulli's equations (Linear). (v) Clairaut's Equations : General and Singular solutions.

Orthogonal Trajectories.

Second order linear equations : Second order linear differential equations with constant coefficients. Euler's Homogeneous equations.

Books Recommended

1. Shanti Narayan, Integral Calculus.
2. T.M.Apostol, Calculus (Vol. I).
3. G.F. Simmons, Differential Equations with Applications and Historical Notes.

GE 3: Algebra
Subject Code: MATH 03GE3
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Complex Numbers: De Moivre's Theorem and its applications. Exponential, Sine, Cosine and Logarithm of a complex number. Definition of e^z, Inverse circular and Hyperbolic functions.

Theory of Equations: Fundamental Theorem of Algebra. Polynomials with real coefficients: Descarte's Rule of sign and its applications. Relation between roots and coefficients. Symmetric functions of roots, Transformations of equations. Solution of a cubic and biquadratic.

Introduction of Group Theory: Definition and examples. Elementary properties using definition of Group. Subgroup, Quotient group, Normal subgroup, Homomorphism and isomorphism.

Rings and Integral Domains: Definition and examples. Subrings and ideals. Quotient ring. Homomorphism ans isomorphism of rings.

Fields: Defnition and examples.

Vector (Linear) space over a field. Subspaces. Linear combinations. Linear dependence and independence of a set of vectors. Linear span. Basis. Dimension. Replacement Theorem. Extension theorem. Deletion theorem.

Row Space and Column Space of a Matrix. Rank of a matrix. Rank(AB) $\leq$ min(Rank A; Rank B).

System of Linear homogeneous equations: Solution space of a homogeneous system and its dimension. System of linear non-homogeneous equations: Necessary and sufficient condition for the consistency of the system. Method of solution of the system of equations.

Linear Transformation (L.T.) on Vector Spaces: Null space. Range space. Rank and Nullity, Sylvester's law of Nullity. Inverse of Linear Transformation. Non-singular Linear Transformation. Change of basis by Linear Transformation. Vector spaces of Linear Transformation.

Characteristic equation of a square matrix. Eigen-value and Eigen-vector. Invariant subspace. Cayley- Hamilton Theorem. Simple properties of Eigen value and Eigen vector.

Books Recommended

1. S. Kumaresan, Linear Algebra: A Geometric Approach, PHI.
2. Rao, Bhimashankaran: Linear Algebra, HBA (TRIM)
3. J.B. Fraleigh, First Course in Abstract Algebra, Narosa.

GE 4: Analytical Geometry
Subject Code: MATH 04GE4
Credits: 6 (Theory-5, Tutorial-1)
Contact Hours per Week: 6 (5 Theory lectures + 1 Tutorial)

Analytical Geometry of two dimensions: Transformations of Rectangular axes: Translation, Rotation and their combinations. Invariants.

General equation of second degree: Reduction to canonical forms and Classification.

Pair of straight lines: Condition that the general equation of 2nd degree may represent two straight lines. Points of intersection of two intersecting straight lines. Angle between two lines. Equation of bisectors of angles. Equation of two lines joining the origin to the points in which a line meets a second degree curve.

Equations of pair of tangents from an external point, chord of contact, poles and polars in case of general conic : Particular cases for Parabola, Ellipse, Circle and Hyperbola.

Polar equation of straight lines and circles. Polar equation of a conic referred to a focus as pole. Equation of chord joining two points. Equations of tangent and normal.

Analytical Geometry of three dimensions: Rectangular Cartesian co-ordinates: Distance between two points. Division of a line segment in a given ratio. Direction cosines and direction ratios of a straight line. Projection of a line segment on another line. Angle between two straight lines.

Equation of a Plane: General form. Intercept and Normal form. Angle between two planes. Signed distance of a point from a plane. Bisectors of angles between two intersecting planes.

Equations of Straight line: General and symmetric form. Distance of a point from a line. Shortest distance between two skew-lines. Coplanarity of two straight lines.

Sphere and its tangent plane.

Right circular cone and right circular cylinder. Familiarity with conicoids. Spherical and cylindrical coordinates.

Books Recommended:

1. S.L. Loney, The Elements of Coordinate Geometry, McMillan.

2. J.T. Bell, Elementary Treatise on Coordinate Geometry of Three Dimensions, McMillan.

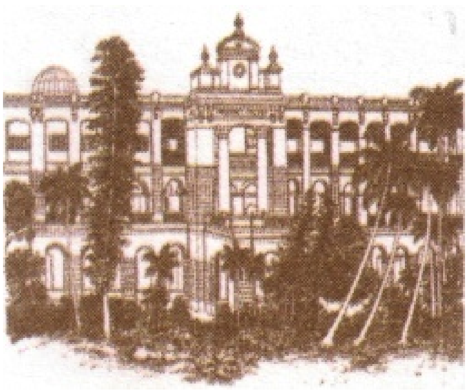
PRESIDENCY UNIVERSITY

DEPARTMENT OF MATHEMATICS

Syllabus for two-year M.Sc. Programme in Mathematics
(effective from the academic session 2021-22)

EXCELLENCE SINCE 1817

PRESIDENCY UNIVERSITY
KOLKATA



Department of Mathematics
(Faculty of Natural and Mathematical Sciences)
Presidency University
Hindoo College (1817-1855), Presidency College (1855-2010)
86/1, College Street, Kolkata - 700 073
West Bengal, India

Course Structure for two-year M.Sc. Programme in Mathematics (with effect from the academic session 2021-22)
Semester-wise distribution of Courses

Semester	Paper Code	Name of the Courses	Page Number	Full Marks	Credit Point	Classes per week	Course Type †
I	MATH0701	Algebra - I	3	50	4	4 hr	T
	MATH0702	Topology - I	4	50	4	4 hr	T
	MATH0703	Ordinary Differential Equations	5	50	4	4 hr	T
	MATH0791	Classical Mechanics	6	50	4	4 hr	S
	MATH0792	Complex Analysis	7	50	4	4 hr	S
		Total		**250**	**20**	**20 hr**	
II	MATH0801	Algebra - II	8	50	4	4 hr	T
	MATH0802	Geometry - I	9	50	4	4 hr	T
	MATH0803	Operations Research	10	50	4	4 hr	T
	MATH0891	Measure and Probability	11	50	4	4 hr	S
	MATH0892	Mathematical Methods - I and Graph Theory	13	50	4	4 hr	S
		Total		**250**	**20**	**20 hr**	
III	MATH0901	Partial Differential Equations	14	50	4	4 hr	T
	MATH0902	Functional Analysis	15	50	4	4 hr	T
	MATH0903	Elective - I (E - I) *	2	50	4	4 hr	T
	MATH0991	Mathematical Methods - II and Number Theory	16	50	4	4 hr	S
	MATH0992	Project **	2	50	4	4 hr	S
		Total		**250**	**20**	**20 hr**	
IV	MATH1001	Algebra - III	18	50	4	4 hr	T
	MATH1002	Dynamical Systems	19	50	4	4 hr	T
	MATH1003	Elective - II (E - II) *	2	50	4	4 hr	T
	MATH1091	Mathematical Computing with Python	21	50	4	4 hr	S
	MATH1092	Dissertation **	2	50	4	4 hr	S
		Total		**250**	**20**	**20 hr**	
		Grand Total		**1000**	**80**		

† In Course Type, 'T' stands for Theory and 'S' stands for Sessional papers. The methods of assessments for Theory and Sessional papers are as follows:

- **Theory: Internal Assessment (15 marks) + Semester Examination (35 marks)**
- **Sessional: Continuous evaluation throughout the semester.**

Options available for Elective - I and Elective - II Courses*

Elective	Course ID	Name of the Courses	Page Number	Full Marks	Credit Point	Classes per week
I	MATH0903A1	Topology-II	23	50	4	4 hr
	MATH0903A2	Advanced Complex Analysis	24	50	4	4 hr
	MATH0903B1	Special Theory of Relativity	25	50	4	4 hr
	MATH0903B2	Qualitative Theory of Planar Vector Fields - I	26	50	4	4 hr
	MATH0903B3	Advanced Operations Research - I	27	50	4	4 hr
	MATH0903B4	Mathematical Biology - I	28	50	4	4 hr
	MATH0903B5	Advanced Numerical Analysis - I	29	50	4	4 hr
II	MATH1003A1	Operator Algebra	30	50	4	4 hr
	MATH1003A2	Geometry - II	31	50	4	4 hr
	MATH1003A3	Abstract Harmonic Analysis	32	50	4	4 hr
	MATH1003B1	General Theory of Relativity and Cosmology	33	50	4	4 hr
	MATH1003B2	Qualitative Theory of Planar Vector Fields - II	34	50	4	4 hr
	MATH1003B3	Advanced Operations Research - II	35	50	4	4 hr
	MATH1003B4	Mathematical Biology - II	36	50	4	4 hr
	MATH1003B5	Advanced Numerical Analysis - II	37	50	4	4 hr

***N.B. : In E - I and II, exactly *one* from 'MATH0903AX & MATH1003AY' and exactly *one* from 'MATH0903BX & MATH1003BY' will be offered.**

Course Structure

Options available for Project & Dissertation**

Topics for project and dissertation include, but are not limited to, the following:

Lie groups, Lie algebras, Representation Theory, Compact Quantum Groups and Quantum Symmetry, Qualitative Theory of Differential Equations, Dynamical Systems, Complex Dynamics, Ergodic Theory, Riemann Surfaces, Algebraic Graph Theory, Domination in Graphs, Mathematical Cryptography, Cyber Security and Mathematics, Data Science and Analysis with Python, Special Theory of Relativity, General Theory of Relativity, Astrophysics and Cosmology, Theoretical and Observational Cosmology, Mechanics.

Algebra - I

Semester : I	Course Type : T
Course ID : MATH0701	Full Marks : 50

Course Structure

- Group Theory: Review of normal subgroups, quotient groups, and isomorphism theorems; Group actions with examples, orbits and stabilisers, class equations and applications; Lagrange's, Cayley's, Cauchy's and Sylow's theorems in the language of group actions; Symmetric and alternating groups, simplicity of A_n; Direct products and free Abelian groups; Semi-direct products; Composition series, exact sequences; Solvable and nilpotent groups. Free groups; Free products, amalgamated free products, HNN extensions, wreath products.
- Ring Theory: Review of integral domains, ideals, quotient rings and isomorphism theorems, prime and maximal ideals, product of rings, prime and maximal ideals in quotient rings and in finite products, Chinese remainder theorem, field of fractions, irreducible and prime elements, UFD, PID, ED; Polynomial rings, division algorithm, irreducibility criteria, Gauss' theorem; Noetherian rings, Hilbert's basis theorem.

References

[1] D. S. Dummit and R. M. Foote, *Abstract Algebra*, Wiley.

[2] S. Lang, *Algbera*, Springer.

[3] T. W. Hungerford, *Algebra*, Springer.

[4] N. S. Gopalakrishnan, *University Algebra*, Wiley.

[5] Michael Artin, *Algebra*, Prentice Hall.

[6] J. J. Rotman, *An Introduction to the Theory of Groups*, Springer.

[7] D. S. Malik, John M. Mordeson and M. K. Sen, *Fundamentals of Abstract Algebra* McGraw-Hill.

[8] Mahima Ranjan Adhikari and Avishek Adhikari, *Basic Modern Algebra with Applications*, Springer.

[9] Joseph A Gallian, *Contemporary Abstract Algebra*, Brooks/Cole Cengage Learning.

Topology - I

Semester : I	Course Type : T
Course ID : MATH0702	Full Marks : 50

Course Structure

- Topological Spaces, Subspace Topology, open and closed sets, neighbourhoods, limit points, interior and closure of a set, dense sets, base and subbase.
- Countability axioms, continuous maps and homeomorphisms.
- Compactness and connectedness, components, path connectedness, locally compact spaces, locally connected spaces, product topology.
- Seperation axioms, regular, completely regular and normal spaces, Urysohn's lemma, Tietz's extension theorem, Urysohn's metrization theorem (statement only), Tychonoff theorem, one-point compactification.
- Topology of pointwise convergence, topology of compact convergence, compact-open topology.
- Quotient spaces with examples (like torus, G/H, Klein's bottle, projective spaces, wedge sum of topological spaces etc.), homotopy, deformation retract, strong deformation retract, contractible spaces.
- Homotopic paths and fundamental group π_1, simply connected topological spaces.
- Covering spaces with examples, path lifting property, homotopy lifting property, computation of $\pi_1(S^1)$, lifting criterion (statement only), deck transformations and properly discontinuous group actions, construction of Universal cover, Galois correspondence for covering spaces.

References

[1] G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill Education.

[2] M. A. Armstrong, *Basic Topology*, Springer.

[3] J. Dugundji, *Topology*, McGraw-Hill Inc., US.

[4] J. Munkres, *Topology, A first course*, Pearson.

[5] J. L. Kelley, *General Topology*, Springer.

[6] J. Munkres, *Elements of Algebraic Topology*, CRC Press.

[7] A. Hatcher, *Algebraic Topology*, Cambridge University Press.

[8] G. E. Bredon, *Topology and Geometry*, Springer.

[9] J. J. Rotman, *Introduction to Algebraic Topology*, Springer.

Ordinary Differential Equations

Semester : I	Course Type : T
Course ID : MATH0703	Full Marks : 50

Course Structure

- Initial value problems, The Fundamental Existence and Uniqueness Theorem, Maximal interval of existence.
- Linear second order ordinary differential equation with variable coefficients: Recapitulation of the basic theory; Separation theorem and Comparison theorem with applications. Exact equations and self-adjoint operator. Boundary value problems and Lagrange identity. Boundary value problems and Green's functions; Construction of Green's functions, properties and applications. Sturm-Liouville Problems; Eigenfunctions expansion, orthogonality of eigenfunctions, completeness of the eigenfunctions.
- Special Functions: Recapitulation of singular points, points at infinity, series solution and Frobenius method. Hypergeometric equation and functions; Confluent hypergeometric functions and properties with applications. Hermite polynomials. Bessel's functions of first and second kinds, normal form of the Bessel's equation, orthogonality of Bessel functions, Bessel-Fourier series. Legendre equation, Legendre functions, orthogonality of Legendre functions and Legendre series.
- Basic introduction to autonomous systems, phase portraits, isoclines, critical points, stability of the critical points, linearization about a critical point.
- Visualization of some solutions using any mathematical application software like Matlab/Maple/ Mathematica/Python.

References

[1] Lawrence Perko, *Differential Equations and Dynamical Systems*, Springer.

[2] G. F. Simmons, *Differential Equations with applications and historical notes*, CRC Press.

[3] A. C. King, J. Billingham and S. R. Otto, *Differential Equations*, Cambridge University Press.

[4] G. Birkhoff, G-C Rota, *Ordinary Differential Equations*, Wiley and Sons.

[5] Carmen Chicone, *Introduction to ordinary differential equations*, Springer-New York.

[6] R. P. Agarwal and D. O'Regan, *Introduction to ordinary differential equations*, Springer.

[7] E. A. Coddington and N. Levinson, *Theory of ordinary differential equation*, McGraw Hill.

[8] A. Chakraborty, *Elements of ordinary differential equations and special functions*, New Age India International.

Classical Mechanics

Semester : I	Course Type : S
Course ID : MATH0791	Full Marks : 50

Course Structure

- Review of Newtonian mechanics for a single particle and a system of particles; simple illustrations of Newton's equation of motion.
- Constraints and their classification, degrees of freedom, generalized coordinates, D' Alembert's principle, Lagrange's equation of motion for a system of holonomic constraints using D' Alembert's principle (differentiable principle) and Hamilton's principle (integral principle); Applications of the Lagrangian formulation; Conservation theorems; Central force problem.
- Hamilton's equations of motion; cyclic coordinates and their consequences, Routhian, Canonical transformations, Examples of canonical transformations; Poisson's brackets; Liouville's theorem; Hamilton Jacobi theory; Action angle variables; Small oscillations; Noether's theorem.
- Canonical perturbation theory; Preliminaries of rigid body dynamics, Euler's angles.
- Visualization of some dynamical problems using any mathematical application software like Matlab/Maple/Mathematica/Python.

References

[1] H. Goldstein, *Classical Mechanics*, Addison-Wesley Publishing Company.

[2] N. C. Rana and P. S. Joag, *Classical Mechanics*, Tata McGraw-Hill Education.

[3] L. D. Landau and E. M. Lifshitz, *Mechanics*, Butterworth Heinemann.

[4] S. T. Thornton and J. B. Marion, *Classical Dynamics of Particles and Systems*, Belmont, CA : Brooks/Cole.

[5] E. T. Whittaker, *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies with an introduction to the problem of three bodies*, Cambridge University Press.

[6] R. P. Feynmann, R. B. Leighton and M. Sands, *The Feynman Lectures on Physics: Vol 1, Vol 2, Vol 3* [1], Addison-Wesley Publishing Company.

[1] These books are optional for reading, however, we kept them in the list because we are quite sure that if you start reading them you will a lot.

Complex Analysis

Semester : I	Course Type : S
Course ID : MATH0792	Full Marks : 50

Course Structure

- Holomorphic functions and the Cauchy-Riemann equations.
- Power series, Analytic Functions, Exponential, Logarithmic and Trigonometric functions, Branch of a complex logarithm.
- Complex integration, Goursat's theorem, Cauchy's integral formula, power series representation, zeros of an analytic function, Liouville's theorem, index of a closed curve, homotopy version of Cauchy's theorem, invariance of integrals under homotopy, Different versions of Cauchy's theorem using homotopy.
- Identity theorem of holomorphic functions, Morera's theorem, sequence of holomorphic functions.
- Classification of singularities, meromorphic functions and residue calculus, Laurent series, contour integration.
- Argument principle, Rouché's theorem, open mapping theorem, maximum modulus principle.
- Möbius transformation, classification of Möbius transformations (elliptic, hyperbolic, parabolic), conformal mapping, Schwarz lemma, conformal automorphisms of disc, upper half plane, complex plane, Riemann sphere.
- Space of continuous functions, normal families, Arzela-Ascorli theorem, compactness and convergence in the space of analytic functions, Montel's theorem, space of meromorphic functions, Riemann mapping theorem.
- (Optional) Infinite product and Weierstrass factorization theorem.
- (Optional) Little Picard theorem and Great Picard theorem.

References

[1] J. B. Conway, *Functions of One Complex Variable*, Narosa Publishing House.

[2] E. M. Stein and R. Shakarchi, *Complex Analysis*, Princeton University Press.

[3] L. V. Ahlfors, *Complex Analysis*, McGraw-Hill Education.

[4] T. W. Gamelin, *Complex Analysis*, Springer.

[5] W. Rudin, *Real and Complex Analysis*, McGraw-Hill Education.

[6] S. G. Krantz, *Complex Analysis: The Geometric Viewpoint*, The Mathematical Association of America.

Algebra - II

Semester : II	Course Type : T
Course ID : MATH0801	Full Marks : 50

Course Structure

- Quick review of Linear Algebra: Vector spaces, linear transformation, matrix of a linear transform, Dual space and double dual.
- Inner-product spaces, Gram-Schmidt orthogonalisation, bi-linear forms, definition of unitary, hermitian, normal, real symmetric and orthogonal linear transformations, spectral theorems; multi-linear forms, alternating forms
- Modules over commutative rings, examples: vector spaces, commutative rings, $\mathbb{Z}$ modules, $F[X]$-modules; submodules. Quotient modules, homomorphisms, isomorphism theorems, $Hom_R(M, N)$ for R-modules M, N, generators and relations for modules, direct products and direct sums, direct summands, free modules, finitely generated modules.
- Field Theory: Field extensions, finite and algebraic extensions, algebraic closure, splitting fields, normal extensions, separable, inseparable and purely inseparable extensions, finite fields, ruler and compass constructions.

References

[1] D. S. Dummit and R. M. Foote, *Abstract Algebra*, Wiley.

[2] S. Lang, *Algbera*, Springer.

[3] T. W. Hungerford, *Algbera*, Springer.

[4] K. Hoffman and R. Kunze, *Linear Algebra*, Prentice-Hall, Inc.

[5] Mahima Ranjan Adhikari and Avishek Adhikari, *Basic Modern Algebra with Applications*, Springer.

Geometry - I (Differential Geometry)

Semester : II	Course Type : T
Course ID : MATH0802	Full Marks : 50

Course Structure

- Manifolds, smooth structure, smooth manifolds with examples ($\mathbb{R}^n, \mathbb{C}^n$, $\mathbb{S}^n$, $\mathbb{R}P^n$, $\mathrm{GL}(n, \mathbb{R})$, product manifolds etc.), smooth mappings and diffeomorphisms with examples.
- Tangent and cotangent spaces, Jacobian matrix, tangent and cotangent bundles; vector fields, integral curves and Lie brackets, flow of a vector field.
- Submanifolds; regular and critical points of a smooth map, immersion, submersion and embeddings. Differential forms and exterior derivatives.
- Riemannian metric and Riemannian manifolds, length of a smooth curve in a Riemannian manifold, Isometries. Affine connections and covariant derivative, parallel transport, Riemannian connection.
- Geodesics and geodesic flow, the exponential map, normal neighbourhood, connected Riemannian manifolds as metric spaces, geodesics minimizing distance locally, Hopf-Rinow theorem.
- Some model spaces like n-sphere S^n, Poincaré upper half plane $\mathbb{H}^2$, disc model of the Poincaré upper half plane, the hyperbolic n-space $\mathbb{H}^n$.
- Torsion tensor field and Riemannian curvature tensor field, the structural equations and its applications. Sectional curvature of a Riemannian manifold, sectional curvature of $\mathbb{R}^n, \mathbb{S}^n, \mathbb{H}^n$. Riemannian manifolds of constant sectional curvature.

References

[1] F. W. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, Springer.

[2] N. J. Hicks, *Notes on Differential Geometry*, Van Nostrand.

[3] J. L. Dupont, *Differential Geometry*, Aarhus Universitet Matematisk Institut, (https://data.math.au.dk/publications/ln/1993/imf-ln-1993-62.pdf).

[4] M. P. Do Carmo, *Riemannian Geometry*, Birkhäuser.

[5] Gallot, Hulin, Lafontaine, Riemannian Geometry, Universitext-Springer.

[6] J. M. Lee, *Riemannian Manifolds An Introduction to Curvature*, Springer.

[7] L. Tu, *Differential Geometry*, Springer

[8] S. Helgason, *Differential Geometry, Lie Groups, and Symmetric Spaces*, AMS.

Operations Research

Semester : II	Course Type : T
Course ID : MATH0803	Full Marks : 50

Course Structure

- Introduction to OR: Origin of OR and its definition. Concept of optimizing performance measure, Types of OR problems, Deterministic vs. Stochastic optimization, Phases of OR problem approach – problem formulation, building mathematical model, deriving solutions, validating model, controlling and implementing solution.
- Linear programming: Examples from industrial cases, formulation & definitions, Simplex methods, bounded-variables algorithm, Duality, formulation of the dual problem, primal-dual relationships, Revised simplex algorithm, Sensitivity analysis.
- Transportation problem: mathematical formulation, north-west-corner method, least cost method and Vogel's approximation method for determination of starting basic solution, algorithm for solving transportation problem. Assignment problem: mathematical formulation, Hungarian method for solving assignment problem, Travelling Salesman Problem.
- Integer Programming: Standard form, the concept of cutting plane, Gomory's all integer cutting plane method, Gomory's mixed integer method, Branch and Bound method.
- Nonlinear Programming: Introduction to nonlinear programming, Convex function and its generalization, Unconstrained and constrained optimization, Method of Lagrange multiplier, KKT necessary and sufficient conditions for optimality.
- Queuing Theory: Definitions – queue (waiting line), waiting costs, characteristics (arrival, queue, service discipline) of queuing system, queue types (channel vs. phase), Kendall's notation, Little's law, steady state behaviour, Poisson's Process & queue, Models with examples - M/M/1 and its performance measures; M/M/C and its performance measures; brief about some special models (M/G/1).
- Brief introduction to multi-objective and multi-stage programming, Goal Programming and Dynamic Programming.

References

[1] Frederick S. Hillier, Gerald J. Lieberman, *Introduction to Operations Research*, McGraw Hill Education.

[2] H. S. Taha, *Operations Research*, Pearson Education.

[3] A. Ravindran, Don T. Phillips, James J. Solberg, *Operations Research: Principles and Practice*, Wiley.

[4] Sasieni Maurice, Arther Yaspan, Lawrence Friedman, *OR methods and Problems*, Wiley.

[5] S. D. Sharma, *Operations Research*, Kedar Nath.

[6] John F Shortle, James M Thompson, Donald Gross, Carl M Harris, *Fundamentals of Queueing Theory*, Fifth Edition, Wiley.

[7] T. L. Saaty, *Elements of Queueing Theory, with Applications*, Dover Publications Inc.

[8] B. R. K. Kashyap and M. L. Chaudhry, *Introduction to queueing theory*, Aarkay Publications.

Measure and Probability

Semester : II	Course Type : S
Course ID : MATH0891	Full Marks : 50

Course Structure

Measure Theory

- Algebra, σ-algebra, Monotone Class Theorem, Measure Spaces.
- Outer Measures, Caratheodory Extension Theorem, Pre-measures, Hahn-Kolmogorov Extension Theorem, Uniqueness of the Extension, Completion of a Measure Space.
- Lebesgue Measure and Its Properties.
- Measurable Functions and Their Properties, Modes of Convergence.
- Integration, Monotone Convergence Theorem, Fatou's Lemma, Dominated Convergence Theorem.
- Product Measures, Fubini's Theorem.
- L_p-spaces, Reisz Representation Theorem.
- Signed Measure, The Radon-Nikodym Theorem and Its Applications.
- Fundamental Theorem of Calculus for Lebesgue Integrals.

Probability Theory

- Probability Measure, Probability Space, Continuity Properties of Probability Measure, Random Variables, Probability Distribution of a Random Variable, Functions of a Random Variable and their Probability Distributions.
- Moments, Moment Inequalities (Markov, Chebycheff, Lyapunov and Jensen's inequalities), Moment Generating Function.
- Random Vectors, Probability Distribution of a Random Vector, Functions of Random Vectors and Their Probability Distributions, Independence.
- Characteristic Function and Its Properties, Uniqueness Theorem, Inversion Theorem, Lévy's Continuity Theorem and Bochner's Theorem (Without Proof).
- Sequence of Random Variables, Convergence in Distribution, Convergence in Probability, Almost Sure Convergence, Convergence in rth Mean, Weak and Strong Law of Large Numbers, Borel-Cantelli lemma, Limiting Characteristic Function, Classical Central Limit Theorem, Lindeberg & Lyapunov Central Limit Theorems (Without Proof), Applications of the Central Limit Theorems.

References

[1] T. Tao, *An Introduction to Measure Theory*, American Mathematical Society.

[2] I. K. Rana, *An Introduction to Measure and Integration*, Narosa.

[3] P. R. Halmos, *Measure Theory*, Springer.

[4] H. L. Royden, *Real Analysis*, Pearson.

[5] W. Rudin, *Real and Complex Analysis*, McGraw Hill Education.

[6] P. Billingsley, *Probability and Measure*, Wiley.

[7] A. Gut, *Probability: A Graduate Course*, Springer.

[8] R. G. Laha and V. K. Rohatgi, *Probability Theory*, Dover Publications Inc.

[9] W. Feller, *Introduction to Probability Theory and Its Applications: Vol. 1 and 2*, Wiley.

Mathematical Methods - I and Graph Theory

Semester : II	Course Type : S
Course ID : MATH0892	Full Marks : 50

Course Structure

Mathematical Methods I

- Tensors: Introduction to tensors, tensor algebra.
- Integral Transforms:

  Fourier Transform: Fourier integral theorem, Riemann-Lebesgue lemma, Cosine and sine transforms, inversion theorem, properties of FT with applications, Derivatives, Convolution theorem, convolution of Fourier sine/cosine transform. Application of FT of ODE and PDE.

  Laplace Transform: Functions of exponential order and existence condition for LT. Properties of LT with applications, Inversion of LT, application in solving ODE and PDE. Complex inversion and Bromwich contour integral.

  Mellin Transform: Introduction to Mellin transforms, properties and applications.

  Hankel Transform (if time permits): Introduction, properties and applications.

Graph Theory

- Graphs, Products of Graphs; Connectedness, Trees, Spanning Tree; Degree Sequences: Havel-Hakimi Theorem and its Applications; Connectivity; Eulerian and Hamiltonian graphs: Ore's Theorem, Dirac's Theorem; Clique Number, Chromatic Number: Their Relations: Brooke's Theorem and Perfect Graphs, Domination number, Independence number: Relations and Bounds. Isomorphism of Graphs, Cayley Graphs, Strongly Regular Graphs: Adjacency Matrix of a Graph: Properties and Eigenvalues;
- Visualization of few graph theoretic results using the software SAGEMATH.

References

[1] B. Spain, *Tensor Calculus, a concise course*, Dover Publications, Inc.

[2] L. Brand, *Vector and Tensor Analysis*, John Wiley & Sons.

[3] H. Lass, *Vector and Tensor Analysis*, McGraw-Hill Book Company, Inc.

[4] I. N. Sneddon, *Use of Integral Transforms*, McGraw Hill.

[5] H. G. ter Morsche, J. C. van den Berg, E. M. van de Vrie, *Fourier and Laplace Transforms*, Cambridge University Press.

[6] I. N. Sneddon, *Fourier Transform*, Dover Publications.

[7] R. N. Bracewell, *Fourier Transform and its Applications*, McGraw Hill.

[8] J. L. Schiff, *Laplace Transform Theory and Applications*, Springer.

[9] D. B. West, *Introduction to Graph Theory*, Pearson.

[10] C. Godsil and G. Royle, *Algebraic Graph Theory*, Springer-Verlag.

[11] R. Diestel, *Graph Theory*, Springer.

Partial Differential Equations

Semester : III	Course Type : T
Course ID : MATH0901	Full Marks : 50

Course Structure

- Recapitulation of the basic definition of a general PDE of order m on a n dimensional space, classifications. Formation of PDE, general solution, complete integral and singular solutions. Lagrange's and Charpit's method with geometrical interpretation. General first order linear and nonlinear PDEs, method of characteristics, Cauchy problem, non characteristic condition.
- Second order PDEs, canonical forms and classifications by characteristic, invariance of discriminant.
- Second-order Hyperbolic Equations: One dimensional wave equation and D'Alembert's solution. Spherical Means, Euler-Poisson-Darboux equation, Poisson solution, Kirchoff's solution, Duhamel's principle. Uniqueness of solution: energy methods. Domain of dependence, Range of influence and Causality.
- Second-order Elliptic Equations: Solution by the method of separation of variables and the derivation of the Poisson solution on a disc. Fundamental solution. Mean value Formula, Strong Maximum Principle, Regularity and smoothness of harmonic functions. Liouville's theorem. Green's function and Dirichlet's problem. Green's function derivation with applications in half plane and a disc. Uniqueness of solution.
- Second-order Parabolic Equations: Method of separation of variables. Fundamental solution and heat kernel. Poisson formula. Solution of the inhomogeneous heat equation. Uniqueness of solution: energy methods.
- Visualization of some solutions using any mathematical application software like Matlab/Maple/ Mathematica/Python.

References

[1] L. C. Evans, *Partial Differential Equations*, American Mathematical Society.

[2] I. N. Sneddon, *Elements of Partial Differential Equations*, Dover Publications.

[3] P. Prasad, R. Ravindran, *Partial Differential Equations*, New Age India International Publishers.

[4] V. I. Arnold, *Lectures on Partial Differential Equations*, Springer.

[5] J. Fritz, *Partial Differential Equations*, Springer.

Functional Analysis

Semester : III	Course Type : T
Course ID : MATH0902	Full Marks : 50

Course Structure

- Normed linear spaces, Banach spaces, Examples and elementary properties, Equivalence of norm, Riesz Lemma and its applications, Review of Baire Category Theorem and its consequences regarding the dimension, Bounded linear operators.
- Hahn-Banach Theorem and its consequences, Hahn-Banach separation Theorems, Banach-Steinhaus theorem, Open mapping theorem, closed graph theorem and its applications.
- Dual space, Computing duals of l^p, L^p and $C[0,1]$, reflexive space and its properties.
- Weak and weak* topology, Schur lemma, Banach-Alaoglu Theorem.
- Hilbert spaces, orthonormal sets, projection theorem, Bessel's inequality, Parseval's identity, Riesz representation theorem.
- Bounded operators on a Hilbert space, adjoint of an operator, self-adjoint operator, unitary and normal operators, projection, spectrum and spectral radius of a bounded operator, Compact operator.
- Review of spectral theorem in finite dimensional Hilbert space, Spectral theorem for compact, self-adjoint operators.

References

[1] J. B. Conway, *A course in Functional Analysis*, Springer.

[2] Walter Rudin, *Functional Analysis*, McGraw Hill.

[3] Kosaku Yosida, *Functional Analysis*, Springer.

[4] B. V. Limaye, *Functional Analysis*, New Age International Publisher.

[5] R. Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency.

Mathematical Methods - II and Number Theory

Semester : III	Course Type : S
Course ID : MATH0991	Full Marks : 50

Course Structure

Mathematical Methods II

- Calculus of Variations: The brachistochrone problem, Hamilton's principle, some variational problems from geometry, extrema of functionals, Euler-Lagrange equations, some special cases of the Euler-Lagrange equations.
- Integral Equations: Definition and classifications. Solution by separable kernels. Approximate method and Neumann series. Fredholm alternative theorem. Resolvent kernel and applications. Conversion of IVP and BVP to integral equations, Green's function. Symmetric kernels and bilinear forms. Hilbert-Schimdt theorem and applications. Symmetric integral equation.
- Visualization of some solutions using any mathematical application software like Matlab/Maple/ Mathematica/Python.

Number Theory

- The Arithmetic of $\mathbb{Z}_p$, p a prime, pseudo prime and Carmichael Numbers, Fermat Numbers, Perfect Numbers, Mersenne Numbers.
- Primitive roots, the group of units of $\mathbb{Z}_n$, the existence of primitive roots.
- Quadratic residues and non quadratic residues, Legendre symbol, proof of the law of quadratic reciprocity, Jacobi symbols.
- Primality Testing, Miller-Rabin test, Solovay Strassen test.
- Application of number theory in Cryptography, specially in Public Key Cryptography such as RSA and ElGamal Public Key Cryptographic schemes. Few attacks on RSA PKC, DLP and Diffie Hellman Key Exchange Protocol.
- Visualization of few number theoretic results using the software SAGEMATH.

References

[1] Bruce van Brunt, *The Calculus of Variations*, Springer.

[2] U. Brechtken-Manderscheid, *Introduction to the Calculus of Variations*, Springer Science+Business Media, B.V.

[3] M. L. Krasnov, G. I. Makarenko and A. I. Kiselev, *Problems and exercises in the Calculus of Variations*, Mir Publishers.

[4] Robert Weinstock, *Calculus of Variations with applications to Physics and Engineering*, Dover Publications.

[5] R. P. Kanwal, *Linear Integral Equations: Theory and Techniques*, Birkhauser.

[6] F. G. Tricomi, *Integral Equations*, Dover Publications.

[7] S. G. Mikhlin, *Linear Integral Equations*, Dover Publications.

[8] D. M. Burton, *Elementary Number Theory*, Wm. C. Brown Publishers, Dulreque, Lowa, 1989.

[9] Gareth A Jones and J Mary Jones, *Elementary Number Theory*, Springer International Edition.

[10] Richard A Mollin, *Advanced Number Theory with Applications* CRC Press, A Chapman & Hall Book.

[11] Mahima Ranjan Adhikari and Avishek Adhikari, *Basic Modern Algebra with Applications*, Springer.

[12] Kenneth. H. Rosen, *Elementary Number Theory and Its Applications* AT&T Bell Laboratories, Addition Wesley Publishing Company.

Algebra - III

Semester : IV	Course Type : T
Course ID : MATH1001	Full Marks : 50

Course Structure

- Tensor product of modules: definition, universal property, 'extension of scalars', basic properties and elementary computations.
- Exact sequences of modules: Projective, injective and flat modules (only definitions and examples).
- Noetherian modules, torsion and annihilator submodules, finitely generated modules over PID, structure theorems for modules over PID: existence (invariant factor form & elementary divisor form) and uniqueness, primary decomposition theorem.
- Applications: (a) to modules over $\mathbb{Z}$: fundamental theorem of finitely generated abelian groups; (b) to modules over $F[X]$: Canonical forms - Rational and Jordan canonical forms.
- Galois theory: Galois extensions and Galois groups, fundamental theorem of Galois theory; Examples, explicit computation and applications of Galois theory; Roots of unity, cyclotomic extensions, construction of regular n-gons, solvability by radicals, quintics are not solvable by radicals.

References

[1] D. S. Dummit and R. M. Foote, *Abstract Algebra*, Wiley.

[2] S. Lang, *Algbera*, Springer, GTM.

[3] David A. Cox, *Galois Theory*, Wiley.

[4] Ian Stewart, *Galois Theory*, Chapman & Hall/CRC.

[5] Joseph Rotman, *Galois Theory*, Springer.

Dynamical Systems

Semester : IV	Course Type : T
Course ID : MATH1002	Full Marks : 50

Course Structure

Continuous Dynamical Systems

- Vector Fields and Flows on $\mathbb{R}^n$, Topological (C^0) Conjugacy and Equivalency, Classification of Linear Flows up to C^0 Conjugacy and Equivalency.
- α & ω Limit Sets of an Orbit, Attractors, Periodic Orbits and Limit Cycles.
- Local Structure of Critical Points (The Local Stable Manifold Theorem, The Hartman-Grobman Theorem, The Center Manifold Theorem), Lyapunov Function.
- Periodic Orbits, The Poincaré Map and Floquet Theory, The Poincaré-Benedixson Theorem, Dulac's Criteria.
- Chaotic Attractors, Lyapunov Exponents, Test for Chaotic Attractors.

Discrete Dynamical Systems

- Examples of discrete dynamical systems, iterations of functions, phase portraits, periodic points and stable sets, differentiability and its implications, attracting/repelling/neutral periodic points, graphical analysis, cobweb diagram, Newton's method as an iterative process.
- Circle maps, rotation number, periodic points of circle maps, Poincaré classification theorem, devil's staircase, Denjoy's example.
- Sarkovskii's theorem and Sarkovskii ordering.
- Limit sets and recurrence, topological conjugacy, topological transitivity, topological mixing, Devaney chaos, topological entropy, structural stability.
- Quadratic family and logistic family, symbolic dynamics, subshifts and codes, subshifts of finite type (SFT), Perron-Frobenius theorem, topological entropy and the Zeta function of an SFT.
- Schwarzian derivative and bound on the number of attracting periodic orbits.
- Bifurcation theory, classification of bifurcations, period doubling cascade, chaos at the end of bifurcation diagram.
- Hausdorff measure and Hausdorff dimension, space-filling curve, iterated function system and fractals.
- (Optional) Dynamics of linear maps, the horseshoe map, hyperbolic toral automorphisms.

References

[1] C. Robinson, *An Introduction to Dynamical Systems: Continuous and Discrete*, AMS.

[2] L. Perko, *Differential Equations and Dynamical Systems*, Springer.

[3] C. Robinson, *Dynamical Systems: Stability, Symbolic Dynamics and Chaos*, CRC Press.

[4] R. L. Devaney, *An Introduction to Chaotic Dynamical Systems*, CRC Press.

[5] M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press.

[6] Y. Pesin and V. Climenhaga, *Lectures on Fractal Geometry and Dynamical Systems*, AMS.

[7] A. Katok and B. Hasselblatt, *Introduction to the Modern Theory of Dynamical Systems*, Cambridge University Press.

[8] M. F. Barnsley, *Fractals Everywhere*, Academic Press Professional.

[9] K. Falconer, *Fractal Geometry: Mathematical Foundations & Applications*, Wiley.

Mathematical Computing with Python

Semester : IV	Course Type : S
Course ID : MATH1091	Full Marks : 50

Course Structure

- **Introduction to Basic Computing:** Basics of Instruction Cycle (fetch-execute cycle). Idea of memory, CPU and GPU.
- **Introduction to Programming Languages:** Types of programming languages. Compiled and Interpreted(Scripting) Languages. Statically typed and Dynamically typed programming languages.
- **Introduction to Python:** Downloading and installing Python. Understanding, how to use Python and PIP(Package Installer for Python). Understanding the usage of Python terminal interpreter. Execution of python script (with basic "hello world" program). Installing and using IPython with Jupyter Notebook. (One can use Kaggle or Google Colab)
- **Basics of Python:**
  Learning the fundamentals of Python programming.
  1. Hello World(Printing)
  2. Indentation, Comments
  3. Built In Data-Types: *int, float, complex str, bool, set, dict*
  4. Iterators: *list, range, str*
  5. Control Flow: Sequential, Decision(if-else, nested if-else), Repetition (for-loop, while-loop).
  6. Function: Function definition, Parameters, Arguments, Local variables, Calling a Function, Built-In Python Functions (abs(), any(), bin(), bytes(), chr(), com(), float(), format(), input(), int(), len(), list(), max(), min(), open(), pow(), print(), str(), sum() etc.).
  7. Python Strings: Replace, Join, Split, Reverse, Uppercase, Lowercase, etc. Use of Len(), index(), find(), join() etc.
  8. File Handling: Opening and manipulating text file, binary files, csv file. Basics of folder manipulation.
  9. Basics of OOPs, objects and methods. Custom data types.
- **Packages and Modules:** What is a Python Library? Learn to use the documentation.
  1. Numpy: Fields of usage. Array, ndarray*, dot product of arrays (real and complex), matrix, product of matrix, transpose of matrix, inverting matrix, finding eigenvalues, singular value decomposition*, mathematical functions in numpy.
  2. Matplotlib: Drawing basic graphs. Drawing graphs from data (scatter, line, pi-chart, bar-chart). Reading and manipulating images.
  3. Pandas (if time permits): I/O of different files (csv, excel file). Basics of DataFrame* and Series Object. Basic Data Analysis and cleaning of Data. Conversion of data types, indexing and iteration of data types.
- **Applications in Basic Mathematics:**
  1. Applications in solving linear and nonlinear ODE (*Runge-Kutta method, shooting methods etc.*).
  2. Applications in evaluating single and multiple integrals (*Trapezoidal, Simpson's, Gaussian Quadrature etc.*).
  3. Application in finding roots of non-linear/transcendental algebraic equations (*Bisection method, Newton's method, fsolve etc.*).

4. Applications in Number Theory: Finding Quadratic Residues, Jacobi Symbols, Probabilistic Primality testing such as Solovay Strassen Algorithm.

- **Applications to Data Science and Machine Learning (ML):** What is Data Science? Usefulness of Data Science, Objective of Machine Learning. Idea of test, train dataset. Classification of ML. Supervised and Unsupervised Learning (mentions of reinforcement learning, transfer learning). Linear Regression, Logistic Regression, Decision Tree, idea of Support Vector Machine.
- **Applications to Cyber Security:** Implementations of various cryptographic primitives such as Public key cryptosystem, Signature Scheme, Secret Sharing, Hash function, Stream Ciphers etc.
- Artificial Neural Network. Back Propagation. Different types of neural network (RNN, CNN etc.).

References

[1] Vernon L. Ceder, *The Quick Python Book*, Second Edition, Manning, 2010.

[2] J. C. Bautista, *Mathematics and Python Programming*, Lulu.com, 2014.

[3] Amit Saha, *Doing Math with Python, No Starch Press*, San Francisco, 2015.

[4] Alex Martelli, Anna Ravenscroft, Steve Holden, *Python in a Nutshell*, 3rd Edition, O'Reilly Media, Inc, 2017.

[5] Christian Hill, *Learning scientific programming with Python*, Cambridge University Press, 2015.

[6] Alex Gezerlis, *Numerical Methods in Physics with Python*, Cambridge University Press, 2020.

Options for Elective - I

Topology - II

Semester : III	Course Type : T
Course ID : MATH0903A1	Full Marks : 50

Course Structure

- Brower fixed-point theorem, Borsuk-Ulam theorem, winding numbers and applications.
- Free abelian groups; Free groups, free products, amalgamated free products and HNN - extensions of groups, Seifert-van Kampen theorem, fundamental groups of closed genus-g and other surfaces, K(G,1) spaces.
- Simplicial complexes, chains and boundary homomorphisms, simplicial homology, examples and computations. Hurewicz theorem: H_1 as the abelianisation of π_1 (explicit illustration through $\pi_1(\Sigma_g)$ and $H_1(\Sigma_g)$).
- Singular homology, chain complexes, homotopoy invariance, equivalence of simplicial and singular homology.
- Relative homology, homology long exact-sequences, excision theorem and applications; computation of degrees of maps between spheres, Mayer-Vietoris sequences and applications.
- CW-complexes, cellular homology, computing homology groups of spaces (like S^n, $\mathbb{R}P^n$, $\mathbb{C}P^n$, lens spaces, closed genus-g surfaces, etc.); Betti numbers and Euler characteristics;
- Nets and filters, Rings of continuous functions, Stone-Čech compactification.
- Homology of groups.

References

[1] J. Kelley, *General Topology*, Springer.

[2] J. Dugundji, *Topology*, UBS Publishers.

[3] L. Gillman and M. Jerison, *Rings of Continuous Functions*, Springer.

[4] J. Munkres, *Topology*, Pearson.

[5] R. C. Walker, *The Stone-Čech compactification*, Springer.

[6] J. Munkres, *Elements of Algebraic Topology*, CRC Press.

[7] A. Hatcher, *Algebraic Topology*, CUP.

[8] M. Greenberg, J. Harper, *Algebraic Topology: A First Course*, The Benjamin/Cummings Publishing Company.

[9] G. Bredon, *Topology and Geometry*, Springer.

[10] W. Fulton, *Algebraic Topology: A First Course*, Springer.

Advanced Complex Analysis

Semester : III	Course Type : T
Course ID : MATH0903A2	Full Marks : 50

Course Structure

- Conformal Mappings, Level curves, Survey of elementary mappings, Elementary Riemann surfaces.
- Revision of Compactness and convergence in the space of analytic functions, Convergence on compact subsets, Hurwitz's classical version, Normality, Montel's theorem, Riemann mapping theorem, Schwarz-Christoffel formula.
- Weierstrass spherical convergence theorem, spherical metric, spherical derivative, Marty's theorem, Zalcman's lemma, Bloch's principle, Fundamental normality.
- Weierstrass factorization theorem, Factorization of the Sine function, Gamma function, Riemann Zeta function, Jensen's Formula, Genus and order of an entire function, Hadamard factorization theorem.
- Runge's theorem, Simple connectedness, Mittag-Leffler's theorem.
- Analytic continuation and Riemann surfaces, Schwarz reflection principle, Analytic continuation along a path, Monodromy theorem, Sheaf of germs of analytic functions on an open set, Analytic manifolds, Covering spaces.
- Basic properties of harmonic functions, Harmonic functions on a disk, Subharmonic and Superharmonic functions, Dirichlet problem, Green's functions, Harmonic measure.
- Bloch's Theorem, the little and the great Picard's theorem.

References

[1] J. B. Conway, *Functions of One Complex Variable*, Narosa Publishing House.

[2] E. M. Stein and R. Shakarchi, *Complex Analysis*, Princeton University Press.

[3] L. V. Ahlfors, *Complex Analysis*, McGraw-Hill Education.

[4] T. W. Gamelin, *Complex Analysis*, Springer.

[5] W. Rudin, *Real and Complex Analysis*, McGraw-Hill Education.

[6] S. G. Krantz, *Complex Analysis: The Geometric Viewpoint*, The Mathematical Association of America.

Special Theory of Relativity

Semester : III	Course Type : T
Course ID : MATH0903B1	Full Marks : 50

Course Structure

- Differentiable manifolds, tensor calculus, partial derivative of a tensor, Lie derivative, affine connection, covariant differentiation, introduction to metric and metric tensor.
- Newton's laws and inertial frames, Galilean transformations, Newtonian relativity, The Michelson-Morley experiment, Einstein's thoughts and his postulates of special theory of relativity.
- The relativity of simultaneity, Lorentz transformations; mathematical properties of Lorentz transformations, spacetime invariant, length contraction, time dilation, twin paradox, relativistic addition of velocities.
- Minkowski's spacetime, space-like, time-like and light-like intervals, lightcone; four vectors, geometry of four vectors, proper time, relativistic mass, momentum and energy, equivalence of mass and energy, energy-momentum tensor.

References

[1] R. Resnick, *Introduction to Special Relativity*, John Wiley & Sons.

[2] A. P. French, *Special Relativity*, CRC Press.

[3] S. Banerjee and A. Banerjee, *The Special Theory of Relativity*, PHI.

[4] Ray D'Inverno, *Introducing Einstein's Relativity*, Clarendon Press.

[5] W. Rindler, *Relativity - Special, general and cosmological*, Oxford University Press

[6] Ta-Pei Cheng, *Relativity, Gravitation and Cosmology*, Oxford University Press

Qualitative Theory of Planar Vector Fields - I

Semester : III	Course Type : T
Course ID : MATH0903B2	Full Marks : 50

Course Structure

- Basic Results on the Qualitative Theory of Planar Vector Fields: Flows, Singularities of Vector Fields, Phase Portrait, Limit Sets, Stability, The Poincaré Map and The Poincaré-Benedixson Theory.
- Normal Form Theory: Near-Identity Transformations, Normal Forms for Certain Singularities of Vector Fields.
- Desingularization of Non-elementary Singularities: Homogeneous and Quasi-homogeneous Blow up, Desingularization and the Lojasiewicz Property, Nilpotent Singularities.
- Global Phase Portrait: Infinite Singularities, Poincaré and Poincaré-Lyapunov Compactification, Phase Portraits for Global Flows, Separatrix Configurations.
- Index Theory: Index of Singularities of a Vector Field, Hopf's Theorem, Vector Fields on the Sphere $\mathbb{S}^2$, The Poincaré-Hopf Index Theorem.
- Integrability: First Integrals and Invariants, Integrating Factors, Invariant Algebraic Curves, Exponential Factors, Darboux Theory of Integrability, Prelle-Singer and Singer Results, Examples.

References

[1] F. Dumortier, J. Llibre and J. C. Artés, *Qualitative Theory of Planar Differential Systems*, Springer.

[2] L. Perko, *Differential Equations and Dynamical Systems*, Springer.

[3] V. I. Arnold, *Geometrical Methods in the Theory of Ordinary Differential Equations*, Springer.

[4] X. Zhang, *Integrability of Dynamical Systems: Algebra and Analysis*, Springer.

[5] V. V. Nemytskii and V. V. Stepanov, *Qualitative Theory of Differential Equations*, Princeton University Press.

[6] A. A. Andronov, E. A. Leontovich, I. I. Gordon and A. G. Maier, *Qualitative Theory of Second-Order Dynamic Systems*, Wiley.

[7] A. D. Bruno, *Local Methods in Non-linear Differential Equations*, Springer.

Advanced Operations Research - I

Semester : III	Course Type : T
Course ID : MATH0903B3	Full Marks : 50

Course Structure

- **Deterministic Inventory control Models:** Nature of inventory problems. Structure of inventory systems. Definition of inventory problem. Important parameters associated with inventory problems. Variables in inventory problems. Controlled and uncontrolled variables. Types of inventory systems and inventory policies. Statistical and dynamical inventory problems. Deterministic inventory models / systems. Harris-Wilson model. Economic lot size systems. Sensitivity of the lot size systems. Order level systems and their sensitivity analysis. Order level lot size and their sensitivity studies. Non-constant demand models under (s, q), (t, si) and (ti, si) policies. Power law and linear travel demand situations. Lot size systems with different cost properties: (i) Quantity discounts, (ii) Price-change anticipation, (iii) Perishable goods system. Multi-item inventory models with (i) single linear restriction, (ii) More than one linear restriction, (iii) non-linear restrictions.
- **Sequencing:** Sequencing problems, Solution of sequencing problems, Processing n jobs through two machines, Processing n jobs through three machines, Optimal solutions, Processing of two jobs through m machines, Graphical method of solution, Processing n jobs through m machines.
- **Game Theory and Decisions Making:** Game theory to determine strategic behavior, Elements of decision theory and decision trees, Elements of cooperative and non-cooperative games, Two-person zero-sum game, Bimatrix games and Lemke's algorithm for solving bimatrix games.

References

[1] John A. Muckstadt, Amar Sapra, *Principles of Inventory Management*, Springer.

[2] Sven Axsater, *Inventory Control*, Springer.

[3] Eliczer Nadder, *Inventory Systems*, John Wiley and Sons.

[4] G. Hadley and T. M. Whitin, *Analysis of Inventory Systems*, Prentice Hall.

[5] R. J. Tersine and M. Hays, *Principles of Inventory and Material Management*, Pearson.

[6] A. Ravindran, Don T. Phillips, James J. Solberg, *Operations Research: Principles and Practice*, Wiley.

[7] H. S. Taha, *Operations Research*, Pearson Education.

[8] Frederick S. Hillier, Gerald J. Lieberman, *Introduction to Operations Research*, McGraw Hill Education.

[9] Sasieni Maurice, Arther Yaspan, Lawrence Friedman, *OR methods and Problems*, Wiley.

[10] S. D. Sharma, *Operations Research*, Kedar Nath.

[11] Paul R. Thie, Gerard E. Keough, *An Introduction to Linear Programming and Game Theory*, Wiley-Interscience

Mathematical Biology - I

Semester : III	Course Type : T
Course ID : MATH0903B4	Full Marks : 50

Course Structure

- Mathematical Biology and the Modeling Process: an Overview.
- Qualitative analysis of continuous models: Steady state solutions, stability and linearization, Routh- Hurwitz Criteria, Phase plane methods and qualitative solutions, Lyapunov second method for stability, bifurcations (saddle-node, transcritical, pitchfork and Hopf).
- Continuous growth functions: Malthus growth, logistic growth, Gompertz growth, Holling type growth. One species models: Different growth models for single species, harvesting of species. Two species models: Equilibria and their stability analysis.
- Epidemic Models: Deterministic model of simple epidemic, Infection through vertical and horizontal transmission, General epidemic- Karmac-Mackendric Threshold Theorem, SI, SIR, SIRS models.
- Discrete system: Overview of difference equations, steady state solution and linear stability analysis, Introduction to Discrete Models, Linear Models, Growth models, Decay models, Discrete Prey-Predator model and Epidemic model.

References

[1] H. I. Freedman, *Deterministic Mathematical Models in Population Ecology*, Marcel Dekker, Inc.

[2] M. Kot, *Elements of Mathematical Ecology*, Cambridge University Press.

[3] D. Alstod, *Basic Populas Models of Ecology*, Prentice Hall, Inc., NJ.

[4] J. D. Murray, *Mathematical Biology - I*, Springer and Verlag.

[5] L. Perko, *Differential Equations and Dynamical Systems*, Springer Verlag.

Advanced Numerical Analysis - I

Semester : III	Course Type : T
Course ID : MATH0903B5	Full Marks : 50

Course Structure

- Errors: Sources and propagation.
- Non linear Equations: Recapitulation of Newton's method, convergence and rate of convergence, Aitken's method and convergence criterion. Applications.
- System of linear equations: Triangular factorization, Iterative methods: Gauss Seidel method and its convergence, Successive Over Relaxation method. Applications
- Approximation of functions: Least square methods, polynomial economization.
- Eigenvalue and Eigenfunctions of a matrix: Power methods, Given's method, Householder method and QR factorization.
- Computer programming and code development using C/Python for Given's method, SOR method, Aitken's method, least square method and visualization of graphical results wherever possible.

References

[1] K. Atkinson, *Introduction to Numerical Analysis*, J. Wiley and Sons.

[2] E. Isaacson and H. B. Keller, *Analysis of Numerical Methods*, Dover Publications.

[3] F. B. Hildebrand, *Introduction to Numerical Analysis*, Dover Publications.

[4] J. Stoer, R. Bulirsch, *Introduction to Numerical Analysis*, Springer Science.

[5] W. Cheney, D. Kincaid, *Numerical Mathematics and Computing*, Brooks/Cole.

[6] William H. Press, Brian P. Flannery, Saul Teukolsky, William T. Vetterling, *Numerical Recipes in C: The Art of Scientific Computing*, Cambridge University Press.

Options for Elective - II

Operator Algebra

Semester : IV	Course Type : T
Course ID : MATH1003A1	Full Marks : 50

Course Structure

- Banach Algebra, Examples and elementary properties, Spectrum and its properties, Spectral radius formula, Maximal ideal space, Gelfand transformation.
- C^*-algebra, Examples and properties, approximate identity, Gelfand-Mazur theorem, Gelfand-Naimark theorem and its applications (Continuous functional calculus), States and pure states, GNS construction.
- Strong and weak operator topology in $\mathcal{B}(H)$, Von-Neumann algebra, projections, double commutant theorem, Kaplansky density theorem, factors.

References

[1] Gerard J. Murphy, *C^*-algebras and operator theory*, Elsevier.

[2] Kehe Zhu, *An introduction to operator algebras*, CRC press.

[3] Bruce Blackadar, *Operator algebras: Theory of C^*-algebras and Von-Neumann algebras*, Springer.

[4] K. R. Davidson, *C^*-algebras by example*, Fields Institute Monographs.

[5] E. C. Lance, *Hilbert C^*-modules*, London Mathematical Society.

[6] Richard V. Kadison, John R. Ringrose, *Fundamentals of the Theory of Operator Algebras, Vol. I and II*, AMS.

Geometry - II (Lie groups, Lie algebras, and Symmetric Spaces)

Semester : IV	Course Type : T
Course ID : MATH1003A2	Full Marks : 50

Course Structure

- Lie groups with examples, Lie algebras with examples, Lie algebra of a Lie group, Lie group homomorphisms and its properties, Lie subgroups, one-one correspondence between connected Lie subgroups of a Lie group and Lie subalgebras of the corresponding Lie algebra, closed Lie subgroups, simply connected Lie groups, exponential map and its properties, adjoint homomorphism and its properties, automorphism group of a Lie algebra as a Lie group, homogeneous manifolds with examples, compact Lie algebras.
- Solvable and nilpotent Lie algebras, theorem of Lie and Engel, Killing form of a Lie algebra, semisimple and simple Lie algebras, Cartan subalgebra of a semisimple Lie algebra, root space decomposition, real forms of complex Lie algebras, compact real form, Cartan decomposition of a real semisimple Lie algebra, classical complex Lie algebras.
- Riemannian locally and globally symmetric spaces, group of isometries of Riemannian globally symmetric spaces, Riemannian symmetric pairs and associated Riemannian globally symmetric spaces, orthogonal symmetric Lie algebras, compact connected Lie groups as Riemannian globally symmetric spaces, totally geodesic submanifolds and Lie triple systems.
- Effective orthogonal symmetric Lie algebras of the compact type, noncompact type and Euclidean type; dual of an orthogonal symmetric Lie algebra; irreducible orthogonal symmetric Lie algebras of type I, II, III, and IV; decomposition of an effective orthogonal symmetric Lie algebra into irreducibles; irreducible Riemannian globally symmetric spaces; decomposition of a simply connected Riemannian globally symmetric space into irreducibles.
- Symmetric spaces of the noncompact type and compact type, maximal compact subgroups of connected semisimple Lie groups, restricted roots, the Iwasawa decomposition of a real semisimple Lie algebra and of a connected semisimple Lie group, Hermitian symmetric spaces, bounded symmetric domains as Hermitian symmetric spaces of the noncompact type.
- Simple complex Lie algebras; Dynkin diagrams; exceptional Lie algebras of type $\mathfrak{e}_6, \mathfrak{e}_7, \mathfrak{e}_8, \mathfrak{f}_4, \mathfrak{g}_2$; description of finite order automorphisms of a complex simple Lie algebra, classification of irreducible Riemannian globally symmetric spaces of type I, II, III, and IV.

References

[1] F. W. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, Springer.

[2] S. Helgason, *Differential Geometry, Lie Groups, and Symmetric Spaces*, AMS.

[3] A. Borel, *Semisimple Groups and Riemannian Symmetric Spaces*, TRIM- HBA.

[4] J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Springer.

Abstract Harmonic Analysis

Semester : IV	Course Type : T
Course ID : MATH1003A3	Full Marks : 50

Course Structure

- **Banach Algebra:** Normed algebra, Banach algebra, examples of Banach algebra, resolvent function and its analyticity, spectrum of a point, spectral radius, ideal and maximal ideal of a Gelfand algebra, character space, maximal ideal space with Gelfand topology, Gelfand representation theorem, theory of non-unital Banach algebras.
- **Topological Group:** Basic definition and facts, subgroups, quotient groups, some special locally compact Abelian groups.
- **Measure Theory on Locally Compact Hausdörff Space:** Positive Borel measure, Riesz representation theorem, Complex measure Radon-Nikodym theorem and its consequences, bounded linear functionals on $L_p (1 \leq p \leq \infty)$, the dual space of $C_0(X)$ for a locally compact Hausdörff space X (the Riesz representation theorem).
- **Haar Measure on Locally Compact Group:** Construction of Haar measure, properties of Haar measure, uniqueness of Haar measure (up to multiplicative constant).
- **Basic Representation Theory:** Unitary representations, Schur's lemma, representations of a group and its group algebra, Gelfand-Raikov theorem.

References

[1] Hewitt and Ross, *Abstract Harmonic analysis, Vol. I and II*, Springer-Verlag.

[2] G. B. Folland, *A Course in Abstract Harmonic Analysis*, CRC Press (1995).

[3] L. H. Loomis, *An Introduction to Abstract Harmonic Analysis*, D. Van Nostrand Company Inc.

[4] Bachman and Narici,*Elements of Abstract Harmonic Analysis*, Academic Press, New York.

[5] Y. Katznelson, *An Introduction to Harmonic Analysis*, Dover Publications, Inc.

General Theory of Relativity and Cosmology

Semester : IV	Course Type : T
Course ID : MATH1003B1	Full Marks : 50

Course Structure

- Einstein's motivations for general relativity, the principle of equivalence, the principle of general covariance, gravity as geometry, the metric and metric tensor, Riemann curvature tensor, Bianchi identity, Ricci tensor, Einstein tensor, Weyl tensor, geodesics, Einstein's field equations, cosmological constant; Schwarzschild solution, Birkhoff's theorem, experimental tests of general relativity, introduction to black holes and their geometries.
- The cosmological principle, Friedmann-Lemâtre-Robertson-Walker (FLRW) metric, cosmic dynamics, Friedmann equations, equation of state, evolution of the scale factor, big bang theory, early universe, present accelerating expansion of the universe, dark energy - the biggest mystery, cosmological parameters.
- Introduction to various observational datasets, simple numerical codes (in Mathematica/C/Python) and their role in general relativity and cosmology.

References

[1] W. Rindler, *Relativity - special, general, and cosmological*, Oxford University Press.

[2] S. M. Carroll, *Space-time and Geometry*, Addison Wesley.

[3] Ray D'Inverno, *Introducing Einstein's Relativity*, Clarendon Press.

[4] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of General Theory of Relativity*, John Wiley & Sons.

[5] Misner, Thorne and Wheeler, *Gravitation*, W. H. Freeman and Company.

[6] S. Weinberg, *Cosmology*, Oxford University Press.

[7] Ta-Pei Cheng, *Relativity, Gravitation and Cosmology*, Oxford University Press.

[8] B. F. Schutz, *A first course in General Relativity*, Cambridge University Press.

[9] J. B. Hartle, *Gravity, an introduction to Einstein's General Relativity*, Addison Wesley.

[10] A. K. Raychaudhuri, *Theoretical Cosmology*, Oxford University Press.

[11] B. Ryden, *Introduction to Cosmology*, Addison Wesley.

Qualitative Theory of Planar Vector Fields - II

Semester : IV	Course Type : T
Course ID : MATH1003B2	Full Marks : 50

Course Structure

- The Center and Focus Problem: The Classical Poincaré Center-Focus Problem, Lyapunov Numbers, Normal Forms, The Center Variety.
- Isochronus Centers and Linearization: The Period Function, Isochronous Center, Darboux Linearization.
- Structural Stability: Piexoto's Theorem, Structural Stability of Vector Fields on Open Surfaces.
- Bifurcation Theory: Unfolding Vector Fields, Universal Unfolding, Local Codimension 1 and 2 Bifurcations of Singularities, Andrnov-Hopf Bifurcation, Bifurcation of Limit Cycles, Homoclinic Bifurcations, Melnikov Theory, Equivariant Bifurcations.
- The Cyclicity Problem: Limit Periodic Sets, The Cyclicity for Limit Periodic Sets, The Second Part of Hilbert's 16th Problem, The Finite Cyclicity Conjecture, The Weak Form of Hilbert's 16th Problem.

References

[1] F. Dumortier, J. Llibre and J.C. Artés, *Qualitative Theory of Planar Differential Systems*, Springer.

[2] L. Perko, *Differential Equations and Dynamical Systems*, Springer.

[3] V. I. Arnold, *Geometrical Methods in the Theory of Ordinary Differential Equations*, Springer.

[4] V. G. Romanovski and D.S. Shafer, *The Center and Cyclicity Problems: A Computational Algebra Approach*, Birkhauser.

[5] S. N. Chow, C. Z. Li and D. Wang, *Normal Forms and Bifurcation of Planar Vector Fields*, Cambridge University Press.

[6] M. Golubitsky and I. Stewart, *The Symmetry Perspective*, Birkhauser.

[7] R. Roussarie, *Bifurcations of Planar Vector Fields and Hilbert's Sixteenth Problem*, Birkhauser.

Advanced Operations Research - II

Semester : IV	Course Type : T
Course ID : MATH1003B3	Full Marks : 50

Course Structure

- **Probabilistic Inventory control Models:** Probabilistic demand models. Expected cost. Probabilistic order level systems. Probabilistic order level systems with instantaneous demand. Probabilistic order level systems with uniform demand. Probabilistic order level systems with lead time. Discrete and continuous probability versions of the models. Problems on the two versions of the models. Newspaper boy problem. Spare parts problem. Baking company problem. Equivalence of probabilistic order level systems.
- **Project Scheduling and Network Analysis:** Types of network problems with examples, flows in network, Max-flow min-cut theorem and its application, Introduction and Basic differences between PERT and CPM, Steps of PERT/CPM Techniques, PERT/CPM network Components and Precedence Relationships, Fulkerson's 'i-j' rule, Critical Path analysis, Forward and backward pass methods, Floats of an activity, Project costs by CPM, Probability in PERT analysis, Project Crashing, Time cost Trade-off procedure, Updating of the Project, Resource Allocation.
- **Replacement Models:** Replacement problem, Types of replacement problems, Replacement of capital equipment that varies with time, Replacement policy for items where maintenance cost increases with time and money value is not considered, Money value, Present worth factor, Discount rate, Replacement policy for item whose maintenance cost increases with time and money value changes at a constant rate, Choice of best machine, Replacement of low cost items, Group replacement, Individual replacement policy, Mortality theorem, Recruitment and promotional problems.

References

[1] John A. Muckstadt, Amar Sapra, *Principles of Inventory Management*, Springer.

[2] Sven Axsater, *Inventory Control*, Springer.

[3] Eliczer Nadder, *Inventory Systems*, John Wiley and Sons.

[4] G. Hadley and T. M. Whitin, *Analysis of Inventory Systems*, Prentice Hall.

[5] R. J. Tersine and M. Hays, *Principles of Inventory and Material Management*, Pearson.

[6] A. Ravindran, Don T. Phillips, James J. Solberg, *Operations Research: Principles and Practice*, Wiley.

[7] H. S. Taha, *Operations Research*, Pearson Education.

[8] Frederick S. Hillier, Gerald J. Lieberman, *Introduction to Operations Research*, McGraw Hill Education.

[9] Sasieni Maurice, Arther Yaspan, Lawrence Friedman, *OR methods and Problems*, Wiley.

[10] S. D. Sharma, *Operations Research*, Kedar Nath.

Mathematical Biology - II

Semester : IV	Course Type : T
Course ID : MATH1003B4	Full Marks : 50

Course Structure

- Difference Equation and its Application: Difference Calculus, Linear first – order difference equations, Nonlinear difference equations, Higher order linear difference equations, Systems of difference equations, Stability Theory, Applications.
- An introduction to partial differential equations and diffusion in biology: Functions of several variables: a review; Random motion and diffusion equation; Diffusion equations and some of its consequences
- Partial differential equation models in biology: Population dispersal models based on diffusion, Density-dependent dispersal, Simple solutions: steady states and travelling waves, Homogeneous steady states, Travelling wave solutions.
- Models for development and pattern formation in biological systems: Homogeneous steady states and inhomogeneous perturbations, conditions for diffusion instability, Physical explanation, Extension to higher dimensions and finite domain.

References

[1] J. D. Murray, *Mathematical Biology - II*, Springer and Verlag.

[2] Leach Edelstein-Keshet, *Mathematical Models in Biology*, The Random House/ Birkhauser Mathematics Series.

[3] L. Perko, *Differential Equations and Dynamical Systems*, Springer Verlag.

[4] D. W. Jordan and P. Smith, *Nonlinear Ordinary Equations- An Introduction to Dynamical Systems*, (Third Edition), Oxford University Press.

[5] S. Goldberg, *Introduction to Difference Equations with illustrative examples from Economics, Psychology and Sociology*, 1987, Dover Books on Mathematics.

Advanced Numerical Analysis - II

Semester : IV	Course Type : T
Course ID : MATH1003B5	Full Marks : 50

Course Structure

- Integration: Newton-Cotes method, derivation of Trapezoidal, Simpson's $\frac{1}{3}$ rule and Simpson's $\frac{3}{8}$ rule using N-C formula, Gaussian Quadrature method, Richardson extrapolation.
- Ordinary differential equations: Recapitulation of Runge Kutta methods, Multistep methods: Adams methods (Adam Bashforth, Adam Moulton), Picard's method.
- Partial differential equations: Introduction to finite difference methods:
  1. Heat equation: Explicit finite difference scheme, implicit Crank-Nicholson scheme, errors and stability.
  2. Wave equation and Poisson equations: Forward schemes, Leapfrog method, Lax-Friedrichs method, Lax-Wendroff method, stability analysis, Von-Neumann analysis, the CFL conditions.
- Computer programming and code development using C/Python for Gaussian quadrature, Adam's multistep methods, solving Laplace and Poisson equation by finite difference methods and visualization of graphical results wherever possible.

References

[1] K. Atkinson, *Introduction to Numerical Analysis*, J. Wiley and Sons.

[2] E. Isaacson and H. B. Keller, *Analysis of Numerical Methods*, Dover Publications.

[3] F. B. Hildebrand, *Introduction to Numerical Analysis*, Dover Publications.

[4] J. Stoer, R. Bulirsch, *Introduction to Numerical Analysis*, Springer Science.

[5] W.Cheney, D. Kincaid, *Numerical Mathematics and Computing*, Brooks/Cole.

[6] William H. Press, Brian P. Flannery, Saul Teukolsky, William T. Vetterling, *Numerical Recipes in C: The Art of Scientific Computing*, Cambridge University Press.