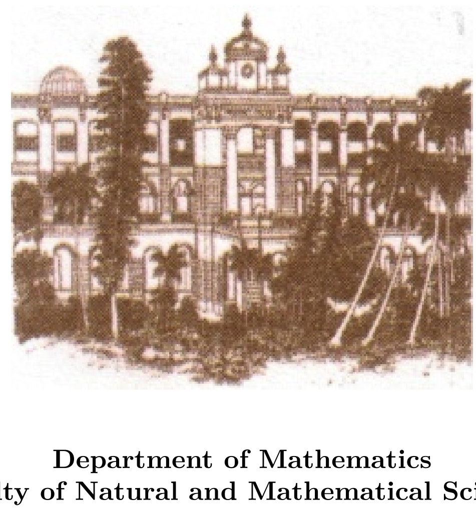
PRESIDENCY UNIVERSITY

DEPARTMENT OF MATHEMATICS

Syllabus for 04 Years Bachelor Programme under
CHOICE BASED CREDIT SYSTEM
for
B.Sc. Honours with Research in Mathematics
(Total Credits: 194)
(effective from 2023-2024 Academic Session)

EXCELLENCE SINCE 1817

PRESIDENCY UNIVERSITY
KOLKATA



Department of Mathematics
(Faculty of Natural and Mathematical Sciences)
Presidency University
Hindoo College (1817-1855), Presidency College (1855-2010)
86/1, College Street, Kolkata - 700 073
West Bengal, India

Programme Outcomes

PO 1 **Developing Analytical and Real-Life Skills:** Students will be able to know the importance of mathematical modelling, simulation and computational methods to solve real world problems. They will be able to model physical, biological, environmental, statistical etc. problems using mathematical knowledge. They will be able to analyse and suggest acceptable real- life solutions using mathematical and data interpretation skills.

PO 2 **Promoting Higher Education:** Students completing this programme will be able to apply their knowledge in Mathematics to construct and develop logical arguments for the solution of complex mathematical problems, describe and formulate mathematical ideas from multiple perspectives. They will be able to explain and apply fundamental concepts of mathematics for solving advanced research problems.

PO 3 **Enhancing Employability in Industry:** Students will be able to use the knowledge acquired in related areas of computer science, statistics and Programming Languages to enhance their employability for government jobs, jobs in software engineering, data science, banking, insurance and investment sectors and in various other public and private enterprises.

PO 4 **Inculcating Innovation and Creativity:** Students will be able to undertake independent research initiatives in mathematics. They will be able to create and hypothesise mathematical results. Will be able to estimate and understand and analyse the limitations of a method and suggest appropriate remedies for tackling such problems.

Course Structure for B.Sc. (4 years) (with research) (w.e.f. August, 2023)
Semester-wise distribution of Courses

Sem	Paper Code	Name of the Courses	Pg. No.	Full Marks	Credit Point	Classes /week	Course Type
I	MATH101C01	Geometry and Introduction to Real Numbers (T)	4	100=30+70	6=5+1	6 hr	Major
	MATH102C02	Algebra (T)	6	100=30+70	6=5+1	6 hr	Major
	MATH104MC01	Differential Calculus (T)	63	100=30+70	6=5+1	6 hr	Minor
	MATH141MDC01	Joy of Numbers 1 (S)	68	100	3	3 hr	MDC
	103AECC01	English Communication/MIL (T/S)		100/50	4		AECC
		Total		**500/450**	**25**	**25 hr**	
II	MATH151C03	Real Analysis - I (T)	8	100=30+70	6=5+1	6 hr	Major
	MATH152C04	Groups and Rings - I (T)	10	100=30+70	6=5+1	6 hr	Major
	153AECC02	English Communication/MIL (T/S)		100/50	4		AECC
	MATH154MC02	Integral Calculus and Differential Equations (T)	64	100=30+70	6=5+1	6 hr	Minor
	MATH191MDC02	Joy of Numbers 2 (S)	69	100	3	3 hr	MDC
	MATH192MDC03	Elementary geometry: The conic sections (S)	70	100	3	3 hr	MDC
		Total		**600/550**	**28**	**28 hr**	
III	MATH201C05	Real Analysis - II and Ordinary Differential Equations (T)	12	100=30+70	6=5+1	6 hr	Major
	MATH202C06	Linear Algebra - I (T)	14	100=30+70	6=5+1	6 hr	Major
	MATH241SEC01	Computer Programming with C (S)	16	50	4	4 hr	SEC
	ENVS204VAC01	Environmental Science		100	3		VAC
	MATH205MC03	Algebra I (T)	66	100=30+70	6=5+1	6 hr	Minor
		Total		**450**	**25**	**25 hr**	
IV	MATH251C07	Sequence and Series of Functions and Metric Spaces (T)	17	100=30+70	6=5+1	6 hr	Major
	MATH252C08	Numerical Methods (T)	19	100=30+70	6=5+1	6 hr	Major
	MATH291SEC02	Latex (S)	21	50	5	5 hr	SEC
	MATH291VAC02	Art of Problem Solving (S)	22	50	3	3 hr	VAC
	MATH255MC04	Algebra II (T)	67	100=30+70	6=5+1	6 hr	Minor
		Total		**400**	**26**	**26 hr**	
V	MATH301C09	Multivariate Calculus (T)	24	100=30+70	6=5+1	6 hr	Major
	MATH302C10	Groups and Rings - II (T)	26	100=30+70	6=5+1	6 hr	Major
	MATH303C11	Probability Theory (T)	28	100=30+70	6=5+1	6 hr	Major
	MATH341SI01	Summer Internship (S)		50	4	-	SI
		Total		**350**	**22**	**18 hr**	
VI	MATH351C12	Complex Analysis (T)	30	100=30+70	6=5+1	6 hr	Major
	MATH352C13	Partial Differential Equations (T)	32	100=30+70	6=5+1	6 hr	Major
	MATH353C14	Optimization Techniques (T)	33	100=30+70	6=5+1	6 hr	Major
	MATH354C15	Mathematical Methods and Graph Theory (T)	35	100=30+70	6=5+1	6 hr	Major
		Total		**400**	**24**	**24 hr**	
VII	MATH401C16	Topology (T)	37	50=15+35	4	4 hr	Major
	MATH402C17	Advanced Ordinary Differential Equations (T)	39	50=15+35	4	4 hr	Major
	MATH403C18	Elective - I (T)	3	50=15+35	4	4 hr	Major
	MATH441C19	Project/Dissertation (S)	3	50	4	4 hr	Major
	MATH442MC05	Research Methodology (S)	44	50	4	4 hr	Minor
		Total		**250**	**20**	**20 hr**	
VIII	MATH451C20	Differential Geometry (T)	41	50=15+35	4	4 hr	Major
	MATH452C21	Classical Mechanics (T)	43	50=15+35	4	4 hr	Major
	MATH453C22	Elective - II (T)	3	50=15+35	4	4 hr	Major
	MATH491C23	Project/Dissertation (S)	3	50	8	8 hr	Major
	MATH492MC06	Research and Publication Ethics (S)	45	50	4	4 hr	Minor
		Total		**250**	**24**	**24 hr**	
		Grand Total		**3200/3100**	**194**		

***N.B. : Marks may vary depending on the choice of AECC.**

Options available for Elective Courses

Course Structure

Elective	Course ID	Name of the Courses	Page Number	Full Marks	Credit Point	Classes per week
I	MATH403C18A1	Advanced Algebra	47	50	4	6 hr
	MATH403C18A2	Advanced Complex Analysis	48	50	4	4 hr
	MATH403C18A3	Number Theory and Cryptography	50	50	4	4 hr
	MATH403C18B1	Operations Research	52	50	4	4 hr
	MATH403C18B2	Tensor Analysis and Integral Transforms	54	50	4	4 hr
	MATH403C18B3	Mechanics	55	50	4	4 hr
II	MATH453C22A1	Measure Theory	56	50	4	4 hr
	MATH453C22A2	Lie Algebra and Representation Theory	57	50	4	4 hr
	MATH453C22A3	Geometric Group Theory	58	50	4	4 hr
	MATH453C22B1	Mathematical Modelling	60	50	4	4 hr
	MATH453C22B2	Special Theory of Relativity	61	50	4	4 hr
	MATH453C22B3	Qualitative Theory of ODEs	62	50	4	4 hr

Course Structure

Options available for Project/Dissertation

Topics for project include, but are not limited to, the following:

Lie groups, Lie algebras, Representation Theory, Compact Quantum Groups and Quantum Symmetry, Dynamical Systems, Complex Dynamics, Ergodic Theory, Riemann Surfaces, Algebraic Graph Theory, Domination in Graphs, Mathematical Cryptography, Cyber Security and Mathematics, Data Science and Analysis with Python, Special Theory of Relativity, General Theory of Relativity, Astrophysics and Cosmology, Theoretical and Observational Cosmology, Mechanics.

Geometry and Introduction to Real Numbers

Semester : I	Course Type : T
Course ID : MATH101C01	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 apply mean value theorems in determining concavity, points of inflections of smooth functions; explain asymptotes, cusps, vertical tangents; trace various functions such as polynomials, rational functions, exponential functions, sine and cosine functions;

CO 2 identify and trace conics represented by general second degree equations in two variables; illustrate the theory of planes, straight lines, spheres, cones, cylinders, surface of revolutions, conicoids in three dimensions;

CO 3 determine the length of smooth curves, area of surface of revolutions, volume of solid of revolutions; compute curvature and torsion of smooth curves in three dimensions.

CO 4 explain the basic properties of real numbers.

Detailed Syllabus

- **Module 1:** Geometry of two dimensions; Orthogonal transformations and invariants, General equation of second degree and its classifications. Geometry of three dimensions; direction cosines of a line, angle between two lines, distance of a point from a line, Equation of a plane, signed distance of a point from a plane, planes passing through the intersection of two planes, angle between two intersecting planes and their bisectors. Parallelism and perpendicularity of two planes. Equations of a line in space, condition for coplanarity of two lines, skew-lines, shortest distance. Spheres, tangent plane of a sphere, orthogonal spheres, cylindrical surfaces, cones, introduction to conicoids.
- **Module 2:** Regular plane curves. Curve tracing. Tangents and normals to the curve. Curvature at a point. Radius of curvature. Concavity and convexity. Point of inflexion. Asymptotes. Singular points of curves; nodes and cusps. Volumes and area by slicing. Parametrized curves; arc length of parametric curves. Special curves: Great circles on spheres. Helix on cylinders. Visualization using any mathematical software.
- **Module 3:** Review of Algebraic and Order Properties of $\mathbb{R}$, Bounded and unbounded subsets; Supremum and Infimum. Least upper bound property of real numbers. Countability of $\mathbb{Q}$ and uncountability of $\mathbb{R}$. The Archimedean property, Construction of $\mathbb{R}$ from $\mathbb{Q}$ by Dedekind's cut. Intervals. Open sets, closed sets. Limit points of a set, Isolated points. Illustrations of Bolzano-Weierstrass theorem for sets. Dense subsets.

References

[1] G. B. Thomas and R. L. Finney, Calculus, Pearson.

[2] M. J. Strauss, G. L. Bradley and K. J. Smith, Calculus, Pearson.

[3] H. Anton, I. Bivens and S. Davis, Calculus, John Wiley.

[4] R. Courant and F. John, Introduction to Calculus and Analysis I & II, Springer.

[5] T. M. Apostol, Calculus I & II, John Wiley.

[6] S. L. Loney, The Elements of Coordinate Geometry, McMillan.

[7] R. J. T. Bell, Elementary Treatise on Coordinate Geometry of Three Dimensions, McMillan.

[8] T. Tao, Analysis II, HBA (TRIM Series).

[9] T. M. Apostol, Mathematical Analysis, Narosa.

[10] W. Rudin, Principles of Mathematical Analysis, McGraw-Hill.

[11] R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, Inc.

Algebra

Semester : I	Course Type : T
Course ID : MATH102C02	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 use De Moivre's theorem in a number of applications to solve numerical problems; determine roots of real and complex polynomials using various methods;

CO 2 explain relations, equivalence relations and partitions;

CO 3 recognize consistent and inconsistent systems of linear equations by the row echelon form of the augmented matrix using rank, and solve consistent system of linear equations; find eigenvalues and corresponding eigenvectors for a square matrix;

CO 4 get a preliminary idea about groups and rings.

Detailed Syllabus

- **Module 1:** Quick review of algebra of complex numbers, modulus and amplitude (principal and general values) of a complex number, polar representation, De-Moivre's theorem and its applications: nth roots of unity.
- **Module 2:** Polynomials with real coefficients and their graphical representation. Relationship between roots and coefficients: Descarte's rule of signs, symmetric functions of the roots, transformation of equations. Solutions of the cubic and bi-quadratic equations by Cardan's and Ferrari's methods. Statement of the fundamental theorem of Algebra. Inequality: The inequality involving AMGMHM, Cauchy-Schwarz inequality.
- **Module 3:** Set, power set, equivalence relations and partitions, partial order, statement of Zorn's lemma. Mappings and functions, injective, surjective and bijective mappings, composition of mappings, invertible mappings. Cardinality of a set, countable and uncountable sets, bijection from the unit interval to unit square using Schroeder-Bernstein theorem, well ordering principle. Divisibility and Euclidean algorithm, congruence relation between integers. Principle of strong and weak mathematical induction and relationship between them. Statement of the fundamental theorem of arithmetic.
- **Module 4:** Elements of $\mathbb{R}^n$ as vectors, linear combination and span of vectors in $\mathbb{R}^n$, linear independence and basis, vector subspaces of $\mathbb{R}^n$, dimension of subspaces of $\mathbb{R}^n$. Linear transformations on $\mathbb{R}^n$ as structure preserving maps, invertible linear transformations, matrix of a linear transformation, change of basis matrix. Scalar product and cross product of vectors in $\mathbb{R}^n$. Adjoint, determinant and inverse of a matrix. Subspaces of $M_n(\mathbb{R})$ (e.g., trace zero matrices and skew symmetric matrices).
- **Module 5:** Elementary row operations: row reductions, elementary matrices, echelon forms of a matrix, rank of a matrix, characterization of invertible matrices using rank. Solution of systems of linear equations $Ax = b$: Gaussian elimination method and matrix inversion method.
- **Module 6:** Definitions and examples: (i) groups, subgroups, cosets, normal subgroups, homomorphisms. (ii) rings, subrings, integral domains, fields, characteristic of a ring, ideals, ring homomorphisms.

References

[1] S. Bernard and J. M. Child, Higher Algebra, Macmillan.

[2] S. K. Mapa, Classical Algebra, Levant.

[3] T. Andreescu and D. Andrica, Complex Numbers from A to Z, Birkhauser.

[4] D. C. Lay, Linear Algebra and its Applications, Pearson.

[5] C. Curtis, Linear Algebra, Springer.

[6] J. B. Fraleigh, A First Course in Abstract Algebra, Pearson.

Real Analysis - I

Semester : II	Course Type : T
Course ID : MATH151C03	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 get an idea about the real number system and its algebraic, order properties; explain important properties of $\mathbb{R}$ and its subsets.

CO 2 explain the concept of sequence of real numbers and its convergence, different criteria of a sequence which makes it a convergent sequence, Cauchy sequence, contractive sequence, subsequence, limit superior and limit inferior etc;

CO 3 elucidate the concept of infinite series of real numbers, its convergence and different tests to check the convergence of a given series;

CO 4 implement the limit and continuity of a function at a point and its geometry, sequential criterion of limit and divergence criterion, various results for the existence of the limit of a function at a point, infinite limits and limits at infinity; uniform continuity, non-uniform continuity criteria, the continuous extension theorem in practical problems.

Detailed Syllabus

- **Module 1:** δ-neighborhood of a point in $\mathbb{R}$, bounded above sets, bounded below sets, bounded Sets, unbounded sets, Supremum and Infimum, sequences, bounded sequence, convergent sequence, limit of a sequence, density of rational (and irrational) numbers in $\mathbb{R}$, intervals. Limit points of a set, isolated points, open sets, closed sets, illustrations of Bolzano-Weierstrass theorem for sets. Monotone sequences, monotone convergence theorem. Subsequences, divergence criteria, monotone subsequence theorem (statement only), Bolzano Weierstrass theorem for qequences. Cauchy sequence, Cauchy's convergence criterion. Contractive sequence, the completeness property of $\mathbb{R}$.
- **Module 2:** Infinite series, convergence and divergence of infinite series, Cauchy criterion, tests for convergence: comparison test, limit comparison test, D'Alembert's ratio test, Cauchy's root test, integral test, alternating series, Leibniz test, Condensation test, Raabe's test, absolute and conditional convergence, Cauchy product, rearrangements of terms, Riemann's theorem on rearrangement of series (statement only).
- **Module 3:** Limits of functions ($\epsilon - \delta$ approach), sequential criterion for limits, divergence criteria. Limit theorems, squeeze theorem, one sided limits. Infinite limits and limits at infinity. Continuous functions, sequential criterion for continuity and discontinuity. Algebra of continuous functions. Continuous functions on an interval, intermediate value theorem, location of roots theorem, preservation of intervals theorem. Uniform continuity, non-uniform continuity criteria, uniform continuity theorem. Lipschitz function. The continuous extension theorem.
- **Module 4:** Differentiability of a function at a point and in an interval, Caratheodory's theorem, Algebra of differentiable functions, chain rule, inverse functions, relative extrema, interior extremum theorem. Rolle's theorem, Mean value theorem, intermediate value property of derivatives - Darboux's theorem. Cauchy's mean value theorem. Applications of mean value theorems to inequalities and approximation of polynomials. Proof of L'Hospital's rule.

- **Module 5:** Taylor's theorem with Lagrange's form of remainder and Cauchy's form of remainder, application of Taylor's theorem to convex functions. Taylor's series and Maclaurin's series expansions of exponential, trigonometric and other functions. Radius of convergence, Cauchy-Hadamard theorem.

References

[1] R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, Inc.

[2] T. Tao, Analysis I, HBA.

[3] W. Rudin, Principles of Mathematical Analysis, McGraw-Hill.

[4] T. M. Apostol, Mathematical Analysis, Narosa.

[5] S. K. Berberian, A First Course in Real Analysis, Springer Verlag.

Groups and Rings - I

Semester : II	Course Type : T
Course ID : MATH152C04	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 link the fundamental concept of groups and symmetries of geometrical objects; explain fundamental properties of permutation groups and alternating groups; elucidate the basic concepts in group theory and applications of Lagrange's theorem; understand free groups;

CO 2 interpret the fundamental concepts in ring theory such as the concepts of ideals, prime ideals, maximal ideals, quotient rings, integral domains etc;

CO 3 illustrate Chinese remainder theorem and its applications;

CO 4 apply above concepts in various fields like Physics, Statistics, Computer Science, Chemistry.

Detailed Syllabus

- **Module 1:** Groups: Groups as symmetries, examples: S_n, A_n, $\mathbb{Z}_n$, D_n, $GL(n, \mathbb{R})$, $SL(n, \mathbb{R})$, $O(n, \mathbb{R})$, $SO(n, \mathbb{R})$ etc., elementary properties of groups, abelian groups.
- **Module 2:** Subgroups and cosets: examples of subgroups including centralizer, normalizer and center of a group, product subgroups; cosets and Lagrange's theorem with applications. Cyclic groups and its properties, classification of subgroups of cyclic groups. Normal subgroups: properties and examples, conjugacy classes of elements, properties of homomorphisms and kernels, quotient groups and their examples. Presentation of a group in terms of generators and relations. Free groups and its universal properties.
- **Module 3:** Properties of S_n, cycle notation for permutations, even and odd permutations, cycle decompositions of permutations in S_n, alternating group A_n, Cayley's theorem. Isomorphism theorems: proofs and applications, isomorphism classes of finite groups of lower order. Cauchy's theorem for finite abelian groups, statements of Cauchy's and Sylow's theorem and applications.
- **Module 4:** Rings: Examples and basic properties of rings, subrings and integral domains. Ideals, algebra of ideals, quotient rings, Chinese remainder theorem.
- **Module 5:** Prime and maximal ideals, quotient of rings by prime and maximal ideals, ring homomorphisms and their properties, isomorphism theorems, field of fractions.
- **Module 6:** Applications of algebra in various fields.

References

[1] I. N. Herstein, Topics in Algebra, John Wiley.

[2] J. B. Fraleigh, A First Course in Abstract Algebra, Pearson.

[3] M. Artin, Algebra, Pearson.

[4] P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, Cambridge Univ. Press.

[5] J. A. Gallian, Contemporary Abstract Algebra, Narosa.

[6] M. R. Adhikari and A. Adhikari, Basic Modern Algebra with Applications, Springer.

[7] J. J. Rotman, An Introduction to the Theory of Groups, Springer.

[8] D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley.

[9] T. W. Hungerford, Algebra, Springer.

[10] M. K. Sen, S. Ghosh, P. Mukhopadhyay and S. K. Maity, Topics in Abstract Algebra, University Press.

Real Analysis - II and Ordinary Differential Equations

Semester : III	Course Type : T
Course ID : MATH201C05	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 illustrate Riemann integrability of bounded functions and algebra of Riemann integrable functions; use the fundamental theorems and mean value theorems of integral calculus in various real world problems; implement the concepts of improper integrals and use various tests (e.g. integral test, comparison test and limit test) for convergence of improper integrals and series;

CO 2 apply the properties of Beta and Gamma functions in various branches of science; identify functions of bounded variations and their properties;

CO 3 determine the existence and uniqueness of various differential equations; solve homogeneous and inhomogeneous linear differential equations using various methods;

CO 4 understand how differential equations arise in real life problems.

Detailed Syllabus

Real Analysis - II

- **Module 1:** Riemann integration. upper and lower sums, Riemann's conditions of integrability. Riemann sum and definition of Riemann integral through Riemann sum. Equivalence of the two definitions. Riemann integrability of monotone and continuous and piecewise continuous functions. Properties of the Riemann integral. Intermediate Value theorem for integrals. Fundamental theorems of Calculus and its consequences. Substitution theorem. Functions of bounded variation and their properties.
- **Module 2:** Improper integrals; proof of integral test for series, convergence of Beta and Gamma functions, Abel's test, Dirichlet's test, Bohr-Mollerup theorem and its consequences.

Ordinary Differential Equations

- **Module 1:** Formation of ordinary differential equations, geometric interpretation, general solution, particular solution.
- **Module 2:** First order equations, existence and uniqueness condition, Lipschitz condition (Statement only), linear first-order equations, exact equations and integrating factors, separable equations, linear and Bernoulli forms. Higher degree equations: general solution and singular solution, p-discriminant and c-discriminant, envelopes of a family of integral curves.
- **Module 3:** Higher order equations: second order equations, general theory and solution of a homogeneous equation, Wronskian (properties and applications), general solution of a non-homogeneous equation, Euler-Cauchy forms, method of undetermined coefficients, normal form, variation of parameters, use of $f(D)$ operator, solution of both homogeneous and inhomogeneous higher order equations (order greater than two).
- **Module 4:** Systems of linear differential equations: basic theory, normal form, homogeneous linear systems with constant coefficients.

References

[1] R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, John Wiley.

[2] T. Tao, Analysis I & II, HBA.

[3] T. M. Apostol, Mathematical Analysis, Narosa.

[4] W. Rudin, Principles of Mathematical Analysis, McGraw-Hill.

[5] C. G. Denlinger, Elements of Real Analysis, Jones & Bartlett.

[6] H. L. Royden, Real Analysis, Pearson.

[7] D. A. Murray, Introductory Course on Differential Equations, Longmans, Green and Co.

[8] S. L. Ross, Differential Equations, John Wiley and Sons.

[9] H. T. H. Piaggio, An Elementary Treatise On Differential Equations, G.Bell And Sons Limited.

[10] G. F. Simmons, Differential Equation with Applications and Historical Notes, CRC Press.

Linear Algebra - I

Semester : III	Course Type : T
Course ID : MATH202C06	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 apply fundamental concepts of vector spaces and linear transformations;

CO 2 determine the rank of a matrix;

CO 3 check the diagonalizability of matrices and linear transformations; determine Rational canonical form and Jordan form of matrices and linear transformations;

CO 4 interpret the geometry of inner product spaces; understand the concept of orthogonal diagonalizability of real symmetric matrices.

Detailed Syllabus

- **Module 1:** Vector spaces, subspaces, algebra of subspaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces, quotient spaces.
- **Module 2:** Linear transformations, null spaces and ranges, rank and nullity of a linear transformation, Sylvester's (rank-nullity) theorem and application, matrix representation of a linear transformation, algebra of linear transformations, invertibility and isomorphisms, change of coordinate matrix.
- **Module 3:** Row space and column space of a matrix, row rank, column rank and rank of a matrix, equality of these ranks, rank of product of two matrices, rank factorisation.
- **Module 4:** Dual Spaces, dual basis, double dual, transpose of a linear transform and its matrix in the dual basis, annihilators.
- **Module 5:** Eigen values and eigen vectors, eigen space of a linear transform, diagonalizability of a matrix, invariant subspaces and Cayley-Hamilton theorem, minimal polynomial for a linear operator, diagonalizability in connection with minimal polynomial, canonical forms (Jordan and Rational).
- **Module 6:** Inner product spaces and norms, Cauchy-Schwarz inequality, orthogonal and orthonormal basis, orthogonal projections, Gram-Schmidt orthogonalisation process and orthogonal complements, Bessel's inequality, the adjoint of a linear operator and its basic properties.
- **Module 7:** Definitions of real symmetric, orthogonal, Hermitian, normal, unitary matrices; spectral theorems for real symmetric matrices.

References

[1] K. Hoffman and R. A. Kunze, Linear Algebra, PHI.

[2] S. Lang, Introduction to Linear Algebra, Springer.

[3] G. Strang, Linear Algebra and its Applications, Academic.

[4] S. Kumaresan, Linear Algebra: A Geometric Approach, PHI.

[5] P. R. Halmos, Finite Dimensional Vector Spaces, Springer.

[6] A. R. Rao and P. Bhimasankaram, Linear Algebra, HBA.

[7] S. H. Friedberg, A. Insel and L. E. Spence, Linear Algebra, Pearson.

[8] C. Curtis, Linear Algebra: An Introductory Approach, Springer (UTM).

Computer Programming with C

Semester : III	Course Type : S
Course ID : MATH241SEC01	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 be acquainted with the basics of programming language in connection with mathematics; understand why C programming is useful for developers;

CO 2 apply loops and decision statements in C programming; apply arrays in C programming; apply the use of pointers in C programming;

CO 3 implement the dynamic memory allocation in C programming; apply functions and pass arguments in C programming; apply and use library functions in C programming; read and write files in C programming;

CO 4 implement in many real life applications.

Detailed Syllabus

- **Module 1:** Introduction to programming in the C language: variables, conditional statements, loops, arrays, functions, recursive programming.
- **Module 2:** Pointer, dynamic memory allocation, linked lists; lists, stacks, queues and trees; file handling.
- **Module 3:** Practicals: searching and sorting algorithms; programs related to number theory, numerical analysis, abstract algebra, geometry etc.

References

[1] B. W. Kernighan and D. M. Ritchie, The C programming Language, PHI.

[2] E. Horowitz and S. Sahani, Fundamentals of Data Structure, Computer Science Press.

Sequence and Series of Functions and Metric Spaces

Semester : IV	Course Type : T
Course ID : MATH251C07	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 apply the concepts of pointwise and uniform convergence of sequences and series of functions and consequences of uniform convergence; apply the concept of power series, termwise integration and differentiation;

CO 2 illustrate several concepts of metric spaces and their properties, e.g. completeness, Bolzano Weierstrass property, compactness, and connectedness;

CO 3 explain the continuity of a function defined on metric spaces and homeomorphism;

CO 4 have ideas about Banach contraction mapping theorem, Arzela-Ascoli theorem on metric spaces and apply in various branches of science.

Detailed Syllabus

Sequence and Series of Functions

- **Module 1:** Point-wise and uniform convergence of sequence of functions, Cauchy criterion for uniform convergence, continuity, derivability and integrability of the limit function of a sequence of functions, series of functions, continuity, derivability and integrability of the sum function, Weierstrass' M-Test, construction of nowhere differentiable continuous maps on $\mathbb{R}$.
- **Module 2:** Power series, radius of convergence, Cauchy-Hadamard theorem, differentiation and integration of power series; Abel's Theorem; Weierstrass' approximation theorem

Metric Spaces

- **Module 3:** Definitions and examples of metric spaces, neighbourhood of a point, interior point and interior of a set, limit point and closure of a set, open set and closed set, bounded sets and diameter of a set, dense subsets, subspaces, equivalent metrics, separable spaces.
- **Module 4:** Convergence of sequences in metric spaces, Cauchy sequences, completeness, completion of a metric space, category properties and Baire category theorem.
- **Module 5:** Continuity of functions, sequential criterion of continuity, uniform continuity, homeomorphisms, isometry.
- **Module 6:** Compactness, Heine-Borel theorem in $\mathbb{R}$, total boundedness, sequential compactness, compactness and continuity.
- **Module 7:** Connectedness, examples of connected metric spaces, connected subsets of R, connectedness and continuity.
- **Module 8:** Contraction mappings, Banach contraction principle, $C(X)$ as a metric space. Arzela-Ascoli theorem, Stone-Weierstrass theorem (statement only).

References

[1] R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, John Wiley.

[2] T. Tao, Analysis I & II, HBA.

[3] T. M. Apostol, Mathematical Analysis, Narosa.

[4] W. Rudin, Principles of Mathematical Analysis, McGraw-Hill.

[5] C. G. Denlinger, Elements of Real Analysis, Jones & Bartlett.

[6] H. L. Royden, Real Analysis, Pearson.

[7] G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill.

[8] S. Kumaresan, Topology of Metric Spaces, Narosa.

[9] I. Kaplansky, Set Theory and Metric Spaces, AMS Chelsea Publishing.

[10] S. Shirali and H. L. Vasudeva, Metric Spaces, Springer.

[11] J. Munkres, Topology, Pearson.

Numerical Methods

Semester : IV Course ID : MATH252C08	Course Type : T Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the concepts of various errors in numerical evaluation of Mathematical Problems; numerically solve transcendental and nonlinear equations, by several iterative methods;

CO 2 apply various methods to interpolate a set of data points; numerically solve any algebraic system of equations, using direct or iterative techniques;

CO 3 numerically solve integrals; numerically solve ordinary differential equations of first order;

CO 4 implement the above methods using C language.

Detailed Syllabus

- **Module 1:** Errors (absolute, relative, round off, truncation).
- **Module 2:** Solution of transcendental and nonlinear equations: bisection method, secant and Regula-Falsi methods, iterative methods, Newton's methods with gradient descent algorithm (steepest descent), convergence and errors of these methods.
- **Module 3:** Interpolation: Lagrange's and Newton's divided difference methods, Newton's forward and backward difference methods, error bounds of these methods, Hermite interpolation.
- **Module 4:** Solution of a system of linear algebraic equations: LU decomposition, Gaussian elimination method, Gauss-Jordan method, Jacobi's and Gauss-Seidel iterative methods and their convergence analysis.
- **Module 5:** Numerical integration: Newton-Cotes formulas; trapezoidal rule, Simpson's rule, composite trapezoidal and Simpson's rule, Bolle's rule, midpoint rule, Simpson's 3/8-th rule, error analysis of these methods.
- **Module 6:** Ordinary differential equations: Euler's method, modified Euler's method and Runge-Kutta method of second and fourth orders.
- **Module 6:** List of Practicals (Using any software):
  - Root finding using bisection, Newton-Raphson, secant and Regula-Falsi method.
  - LU decomposition.
  - Gauss-Jacobi method.
  - Gauss-Seidel method.
  - Interpolation using Lagrange's and Newton's divided differences.
  - Integration using Simpson's Rule.
  - Differentiation using Runge-Kutta method.

References

[1] K. Atkinson, Introduction to Numerical Analysis, John Wiley & Sons.

[2] W. H. Press, S. A. Teukolsky, W. T. Vetterling and B. P. Flannery, Numerical Recipes in C, Cambridge University Press.

[3] R. L. Burden and J. D. Faires, Numerical Analysis, Brooks/Cole.

[4] U. Ascher and C. Greif, A First Course in Numerical Methods, PHI.

[5] J. Mathews and K. Fink, Numerical Methods using Matlab, Pearson.

[6] M. K. Jain, S. R. K. Iyengar and R. K. Jain, Numerical Methods for Scientific & Engineering Computation, New Age International (P) Limited.

Latex

Semester : IV	Course Type : S
Course ID : MATH291SEC02	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand how latex brings ease in mathematical writing;

CO 2 write projects and papers with latex;

CO 3 insert tables, figures, equations in a document; create bibliographies;

CO 4 make easily customizable presentations.

Detailed Syllabus

- **Module 1:** Motivation behind learning Latex; document structure; typesetting text, math modes; tables; figures; equations; referencing;
- **Module 2:** Beamer presentation.

References

[1] H. Kopka and P. W. Daly, Guide to Latex, Addison-Wesley

[2] S. Kottwitz, Latex Beginner's Guide, Packt Publishing Ltd.

Art of Problem Solving

Semester : IV	Course Type : S
Course ID : MATH291VAC02	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 solve and apply a range of problems in linear algebra by using the rank-nullity theorem, Cayley-Hamilton theorem and various diagonalizability criteria among other things;

CO 2 solve and apply a range of problems in group theory as application of Cauchy's theorem and Sylow's theorem and various problems in ring theory involving prime and maximal ideals and corresponding quotients;

CO 3 solve and apply a range of problems in real analysis, in particular on continuity, differentiability and Riemann integration; solve and apply various types of problems in point-set topology and metric spaces;

CO 4 solve and apply various types of problems in (a system of) first-order ODEs, homogeneous and non-homogeneous ODEs and first and second order PDEs.

Detailed Syllabus

- **Module 1:** Linear Algebra: Vector spaces, subspaces, linear independence, basis and dimension, Linear transformations, null spaces and ranges, use of Rank-Nullity theorem, row space and column space of a matrix, row rank, column rank and rank of a matrix, eigenvalues and eigenvectors, eigenspace of a linear transform, diagonalizability of a matrix, application of Cayley-Hamilton theorem, minimal polynomial, diagonalizability in connection with minimal polynomial.
- **Module 2:** Abstract Algebra: Symmetric groups and dihedral groups, group action, application of Cauchy's theorem and Sylow's theorems, simplicity, application of fundamental theorem of finitely generated abelian groups, ideals, maximal and prime ideals, polynomial rings.
- **Module 3:** Analysis (Real Analysis): Sequence, series, point-set topology in real numbers, continuity, uniform continuity, differentiability, sequence of functions and series of functions, Riemann integration, improper integrals, bounded variation, analytic functions and power series.
- **Module 4:** Metric Space: Topology in metric spaces, compactness and connectedness, application of contraction principle.
- **Module 5:** Ordinary Differential Equations and Partial Differential Equations: Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogeneous and non-homogeneous linear ODEs, variation of parameters, first and second order partial differential equations.

References

[1] Berkeley Problems in Mathematics: P N de Souza and J-N Silva.

[2] All the Math You Missed: (But Need to Know for Graduate School): T A Garrity.

[3] Proofs: A Long-Form Mathematics Textbook: J Cummings.

[4] Real Analysis: A Long-Form Mathematics Textbook: J Cummings.

[5] The Stanford Mathematics Problem Book: G Polya and J Kilpatrick.

[6] How to solve it: G Polya.

[7] Solving Mathematical Problems: A Personal Perspective: T Tao.

[8] Calculus: Volume 1 & 2 : T Apostol.

[9] Abstract Algebra: D S Dummit and R M Foote.

[10] Differential Equations with Applications and Historical Notes: G.F. Simmons.

[11] Topology of Metric Spaces: S. Kumaresan.

[12] Differential Equations: Linear, Nonlinear, Ordinary, Partial: King, Billingham, Otto.

[13] Differential Equations and Boundary Value Problems: Computing and Modeling: Edwards, Penney and Calvis.

[14] Differential Equations: S.L Ross.

[15] Linear Algebra Done Right: Sheldon Axler.

Multivariate Calculus

Semester : V	Course Type : T
Course ID : MATH301C09	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the concept of limit and continuity for functions of several variables; be familiar with partial derivatives, directional derivatives; apply important results for differentiable functions, e.g. mean value theorem, inverse and implicit function theorems in higher dimensions;

CO 2 apply the higher order derivatives related results, e.g. Taylor's theorem, maxima and minima theorem; apply Lagrange's multipliers in constrained optimization problems;

CO 3 illustrate the notions of the tangent space, vector field and properties of gradient vector field; understand and apply the line integral of a vector field over piecewise smooth curves;

CO 4 conceptualize double integral over rectangular/non-rectangular region, polar coordinates and conceptualize triple integral in cylindrical and spherical coordinates; understand the change of variable in double and triple integrals; apply Green's theorem, Gauss divergence theorem, Stoke's theorem in determining surface and volume integrals.

Detailed Syllabus

- **Module 1:** Functions of several variables, limit and continuity, partial derivatives, differentiability and total derivatives as matrices, sufficient condition for differentiability, chain rule; gradient vector, directional derivatives, mean value theorem and inequality in higher dimensions, inverse and implicit function theorems.
- **Module 2:** Higher derivatives and Taylor's theorem, maxima and minima, rank theorem constrained optimisation problems, Lagrange's multipliers.
- **Module 3:** Tangent spaces, definition of a vector field, divergence and curl of a vector field, identities involving gradient, curl and divergence; maximality and normality properties of the gradient vector field.
- **Module 4:** Double integrals: (i) over Rectangular region, (ii) over non-rectangular regions, (iii) in polar co-ordinates; triple integrals over: (i) a parallelopiped, (ii) solid regions; computing volume by triple integrals in cylindrical and spherical co-ordinates; change of variables in double integrals and triple integrals with proof.
- **Module 5:** Line integrals, applications of line integrals: mass and work, fundamental theorem for line integrals, conservative vector fields, independence of path. Green's theorem, surface integrals, Stoke's theorem, the divergence theorem.

References

[1] T. Tao, Analysis II, HBA (TRIM Series).

[2] T. M. Apostol, Mathematical Analysis, Narosa.

[3] W. Rudin, Principles of Mathematical Analysis, McGraw-Hill.

[4] T. M. Apostol, Calculus II, John Wiley.

[5] M. Spivak, Calculus on Manifold.

[6] Charles C. Pugh, Real Mathematical Analysis, Springer.

[7] E. Marsden, A. J. Tromba and A. Weinstein, Basic Multivariable Calculus, Springer.

[8] J. Stewart, Multivariable Calculus, Concepts and Contexts; Thomson.

Groups and Rings - II

Semester : V	Course Type : T
Course ID : MATH302C10	Full Marks : 100

Course Structure

Outcomes of the Course

This course will enable the students to:

CO 1 understand group actions and the importance of permutation groups; apply Sylow's theorems in determining groups of a given order; determine groups of automorphisms for finite and infinite cyclic groups;

CO 2 determine all finite abelian groups of a given order;

CO 3 explain some advanced concepts of ring theory such as principal ideal domain, Euclidean domain, unique factorization domain etc;

CO 4 apply Eisenstein's criterion to check irreducibility of polynomials with coefficients in a unique factorization domain.

Detailed Syllabus

- **Module 1:** Groups: Group actions, stabilisers and kernels, orbit-stabiliser theorem and applications, permutation representation associated to a group action, Cayley's theorem via group action, groups acting on themselves by conjugation, class equation and consequences, conjugacy in S_n, p-groups, proof of Cauchy's theorem and Sylow's theorems and consequences, simplicity of A_n
- **Module 2:** Automorphisms of a group, inner automorphisms, group of automorphisms and their computations (in particular for finite and infinite cyclic groups), characteristic subgroups, commutator subgroup and its properties.
- **Module 3:** Direct product of groups, properties of external direct products, $\mathbb{Z}_n$ as external direct product, internal direct products, fundamental theorem of finite abelian groups, fundamental theorem of finitely generated abelian groups (statement only) and its applications.
- **Module 4:** Rings: Polynomial rings over commutative rings, division algorithm and consequences, factorisation of polynomials, reducibility tests, irreducibility tests, Eisenstein's criterion. Some special ideals: Nilradical, Annihilating Ideal, Jacobson radical, Principal Ideal Domains (PID), unique factorisation in Z[x], divisibility in integral domains, irreducibles and primes, Unique Factorisation Domains (UFD), Euclidean Domains (ED), local rings.
- **Module 5:** Applications of algebra in various fields.

References

[1] J. A. Gallian, Contemporary Abstract Algebra, Narosa.

[2] J. B. Fraleigh, A First Course in Abstract Algebra, Pearson.

[3] M. Artin, Abstract Algebra, Pearson.

[4] T. W. Hungerford, Algebra, Springer.

[5] D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley.

[6] P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, CUP.

[7] M. R. Adhikari and A. Adhikari, Basic Modern Algebra with Applications, Springer.

[8] J. J. Rotman, An Introduction to the Theory of Groups, Springer.

[9] C. Musili, Introduction to Rings and Modules, Narosa.

Probability Theory

Semester : V	Course Type : T
Course ID : MATH303C11	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 construct sample spaces associated with random experiments; demonstrate knowledge of random variables and their probability distributions, types of random variables with examples, functions of a random variable and their distributions;

CO 2 illustrate an understanding of a random vector and probability distribution of a random vector;

CO 3 demonstrate knowledge of various modes of convergence, weak and strong law of large numbers along with applications;

CO 4 demonstrate knowledge of the central limit theorem for independent and identically distributed random variables with examples.

Detailed Syllabus

- **Module 1:** Classical Theory and Its Limitations: Random experiment and events, event space, classical definition of probability and its drawback, statistical regularity, frequentist probability.
- **Module 2:** Probability Axioms: Probability measure, probability space, continuity law in probability, exclusion-inclusion formula, conditional probability and Bayes' rule, Boole's inequality, independence of events, Bernoulli trials and binomial law, Poisson trials, probability on finite sample spaces, geometric probability.
- **Module 3:** Random Variables and Their Probability Distributions: Random variables, probability distribution of a random variable, discrete and continuous random variables, some discrete and continuous distributions on $\mathbb{R}$ (Bernoulli, binomial, Poisson; uniform, normal, gamma, Cauchy, exponential and χ^2 -distributions); functions of a random variable and their probability distributions.
- **Module 4:** Moments and Generating Functions: Expectation, moments, measures of central tendency, measures of dispersion, measures of skewness and kurtosis, Markov, Chebycheff & other moment inequalities, probability generating function, moment generating function.
- **Module 5:** Probability Distributions on $\mathbb{R}^n$: Random vectors, probability distribution of a random vector, functions of random vectors and their probability distributions, moments, generating functions, correlation coefficient, conditional expectation, the principle of least squares, regression.
- **Module 6:** Modes of Convergence: Sequence of random variables, almost sure convergence, convergence in probability, convergence in rth mean and convergence in distribution, relations between the convergence concepts, weak and strong law of large numbers, Borel-Cantelli lemmas, applications of the weak law of large numbers.
- **Module 7:** Characteristic Functions: Definition, properties, uniqueness, inversion, continuity theorem, Parseval's relation, Taylor expansion of characteristic functions, Polya's criterion for characteristic functions.
- **Module 8:** Central Limit Theorems: Lindeberg-Levy central limit theorem, Lindeberg condition, Lyapunov condition, Lindeberg-Feller central limit theorem.

References

[1] W. Feller, An introduction to probability theory and its applications I, J. Wiley.

[2] Stirzaker and Grimmett: Probability and Random Processes.

[3] Rohatgi and Saleh: An introduction to probability and statistics, John Wiley.

[4] Allan Gut: An Intermediate Course in Probability, Springer

[5] R B Ash: Basic Probability Theory, Dover Publication

Complex Analysis

Semester : VI	Course Type : T
Course ID : MATH351C12	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 conceptualize *differentiability* - *analyticity* - *holomorphicity* of a complex function;

CO 2 compute complex integrals.

CO 3 learn about singularities and apply techniques of meromorphic functions.

CO 4 illustrate the role of Morera's theorem and to prove Riemann mapping theorem.

Detailed Syllabus

- **Module 1:** Complex numbers, geometric interpretation, stereographic projection, functions of complex variables, mappings, derivatives, holomorphic functions, differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability.
- **Module 2:** Analytic functions and power series, analytic functions are holomorphic, absolute and uniform convergence of power series, examples of analytic functions, exponential function, trigonometric function, complex logarithm.
- **Module 3:** Rectifiable paths, Riemann-Stieltjes integral of a function over an interval, complex integration of functions over a rectifiable path, contours and toy contours, contour integrals and its examples, upper bounds for moduli of contour integrals, Cauchy-Goursat theorem, Cauchy's integral formula, equivalence of analyticity and holomorphicity.
- **Module 4:** Liouville's theorem and the fundamental theorem of algebra. Zeros of analytic functions and identity principle. Morera's theorem. Convergence of sequences and series of analytic functions.
- **Module 5:** Index of a closed curve, homotopy version of Cauchy's theorem, invariance of integrals under homotopy, Different versions of Cauchy's theorem using homotopy.
- **Module 6:** Poles, removable and essential singularity, Riemann's theorem on removable singularities, residue formula, Casorati-Weierstrass theorem, meromorphic functions, Laurent series. The argument principle, the open mapping property of holomorphic functions, maximum modulus principle.
- **Module 7:** Möbius transformation, classification of Möbius transformations (elliptic, hyperbolic, parabolic), conformal mapping, Schwarz lemma, conformal automorphisms of disk, upper half plane, complex plane, Riemann sphere.
- **Module 8:** Space of continuous functions, normal families, Arzela-Ascoli theorem, compactness and convergence in the space of analytic functions, Montel's theorem, space of meromorphic functions, Riemann mapping theorem.

References

[1] J. B. Conway, Functions of One Complex Variable, Narosa Publishing House.

[2] E. M. Stein and R. Shakarchi, Complex Analysis, Princeton University Press.

[3] L. V. Ahlfors, Complex Analysis, McGraw-Hill Education.

[4] T. W. Gamelin, Complex Analysis, Springer.

[5] W. Rudin, Real and Complex Analysis, McGraw-Hill Education.

[6] S. G. Krantz, Complex Analysis: The Geometric Viewpoint, The Mathematical Association of America.

Partial Differential Equations

Semester : VI	Course Type : T
Course ID : MATH352C13	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 formulate many physical phenomena in terms of partial differential equations (PDEs);

CO 2 classify first and second order PDEs and solve them using various techniques;

CO 3 explain the significance of heat equation, wave equation and Laplace equation in various real world problems;

CO 4 visualize solution of PDEs using some mathematical softwares and programming.

Detailed Syllabus

- **Module 1:** Partial differential equations: basic concepts and definitions, mathematical problems, first- order equations: classification, construction and geometrical interpretation; method of characteristics for obtaining general solution of quasi linear equations, canonical forms of first-order linear equations. Method of separation of variables for solving first order partial differential equations. Lagrange's method and its geometric interpretations, Charpit's Method.
- **Module 2:** Classification of second order linear equations as hyperbolic, parabolic or elliptic, reduction of second order linear equations to canonical forms.
- **Module 3:** The Cauchy problem, the Cauchy-Kovalevskaya theorem, one dimensional homogeneous wave equation, heat equation and Laplace equation. Vibrating string with fixed end points, heat equation in a rod, Laplace equation in a rectangle and disc.
- **Module 4:** Fourier series, Fourier coefficients and orthogonality. The Fourier convergence theorem, solution of wave, heat and Laplace equation by the method of separation of variables and Fourier series.
- **Module 5:** Practicals (Using any software): Plotting the integral surfaces and characteristics for a first order PDE. Visualization of solution for the second order canonical PDEs.

References

[1] I. N. Sneddon: Elements of Partial Differential equations, Dover Publications.

[2] E. T. Copson: Partial Differential Equations, CUP.

[3] T. Amarnath, An elementary course in partial differential equations, Narosa.

[4] T. Myint-U and L. Debnath, Linear Partial Differential Equations for Scientists and Engineers, Springer.

Optimization Techniques

Semester : VI	Course Type : T
Course ID : MATH353C14	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 gain insights in managerial decision to choose the best possible course of action to optimize resource allocation of a real-life problem keeping in mind the linear constraints involved: this has useful application in logistics and economical systems;

CO 2 understand the dual nature of real-life problems and how to utilise the duality to solve a given problem more easily;

CO 3 implement game theory in the study of mathematical models of strategic interaction between rational decision-makers and its applications in different fields of social sciences;

CO 4 formulate and solve some real life problems where integer valued outcomes are desirable; formulate and solve network problems arising in real life; solve unconstrained and constrained nonlinear programming problems.

Detailed Syllabus

- **Module 1:** General form of linear programming problems: basic formulation and geometric interpretation, standard and canonical forms. Basic solutions, examples, feasible solutions, degenerate solutions, reduction of a feasible solution to a basic feasible solution; convex set of feasible solutions of a system of linear equations and linear in-equations; extreme points, extreme directions, and boundary points of a convex set, describing convex polyhedral sets in terms of their extreme points and extreme directions, correspondence between basic feasible solution of a system of linear equations and extreme point of the corresponding convex set of feasible solutions.
- **Module 2:** Theory of simplex method, optimality and unboundedness, the simplex algorithm, simplex method in tableau format, artificial variables: two-phase method, Big-M method and their comparison, special cases, Bland's rule, limitations of simplex method, introduction to interior point method, Karmarkar's methods.
- **Module 3:** Duality, formulation of the dual problem, primal-dual relationships, economic interpretation of the dual, revised simplex algorithm, sensitivity analysis.
- **Module 4:** Transportation problem, mathematical formulation, north-west-corner method, least cost method, and Vogel approximation method for determination of starting basic solution, algorithm for solving transportation problem, special cases, assignment problem as special case of transportation problem, mathematical formulation, Hungarian method for solving assignment problem, special cases, travelling salesman problem.
- **Module 5:** Integer Programming: Standard form, the concept of cutting plane, Gomory's all integer cutting plane method, Gomory's mixed integer method.

- **Module 6:** Nonlinear Programming: Introduction to nonlinear programming, convex function and its generalization, unconstrained and constrained optimization, method of Lagrange multiplier, KKT necessary and sufficient conditions for optimality.
- **Module 7:** Network and graph problems: minimal spanning trees, shortest path, flows in networks.
- **Module 8:** Formulation of two person zero sum games, solving two person zero sum games, games with mixed strategies, graphical solution procedure, linear programming solution of games.

References

[1] G. Hadley, Linear Programming, Narosa.

[2] H. Karloff, Linear Programming, Modern Birkhäuser Classic.

[3] D. G. Luenberger and Y. Ye, Linear and nonlinear programming, Springer.

[4] M. J. Osborne and A. Rubinstein, A Course in Game Theory, The MIT Press.

[5] R. Myerson, Game Theory, Harvard University Press.

[6] S. R. Chakravarty, M. Mitra and P. Sarkar, A Course in Cooperative Game Theory, Cambridge University Press.

Mathematical Methods and Graph Theory

Semester : VI	Course Type : T
Course ID : MATH354C15	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 solve variational problems from geometry, extrema of functionals, etc., using the method of calculus of variations; able to convert initial and boundary value problems to integral equations;

CO 2 able to know different techniques in solving various integral equations that frequently arise in real world problems;

CO 3 classify graphs and understand its application in real-life problems;

CO 4 visualize some graph theoretic results using the software SAGEMATH.

Detailed Syllabus

Mathematical Methods

- **Module 1:** Calculus of Variations - The brachistochrone problem, Hamilton's principle, some variational problems from geometry, extrema of functionals, Euler-Lagrange equations, some special cases of the Euler-Lagrange equations.
- **Module 2:** Integral Equations - Definition and classifications of integral equations. Conversion of IVP and BVP to integral equations; Solution of integral equations by separable kernels. Different methods to solve the integral equations.

Graph Theory

- **Module 3:** Graphs, products of graphs; ponnectedness, trees, spanning tree; degree sequences: Havel-Hakimi theorem and its applications; connectivity; Eulerian and Hamiltonian graphs: Ore's theorem, Dirac's theorem; clique number, chromatic number: their relations: Brooke's theorem and perfect graphs, domination number, independence number: relations and bounds. Isomorphism of graphs, Cayley graphs, strongly regular graphs: adjacency matrix of a graph: properties and eigen values;
- **Module 4:** Visualization of few graph theoretic results using the software SAGEMATH.

References

[1] B. van Brunt, The Calculus of Variations, Springer.

[2] U. Brechtken-Manderscheid, Introduction to the Calculus of Variations, Springer Science+Business Media, B.V.

[3] M. L. Krasnov, G. I. Makarenko and A. I. Kiselev, Problems and exercises in the Calculus of Variations, Mir Publishers.

[4] R. Weinstock, Calculus of Variations with applications to Physics and Engineering, Dover Publications.

[5] R. P. Kanwal, Linear Integral Equations: Theory and Techniques, Birkhauser.

[6] F. G. Tricomi, Integral Equations, Dover Publications.

[7] S. G. Mikhlin, Linear Integral Equations, Dover Publications.

[8] D. B. West, Introduction to Graph Theory, Pearson.

[9] C. Godsil and G. Royle, Algebraic Graph Theory, Springer-Verlag.

[10] R. Diestel, Graph Theory, Springer.

Topology

Semester : VII	Course Type : T
Course ID : MATH401C16	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 explain fundamental concepts of topological spaces such as open sets, closed sets, neighbourhoods, limit points, dense subsets, base, subbase, continuous maps, homeomorphisms, countability axioms, separation axioms, compactness, connectedness, path connectedness, components, path components, locally compact spaces, locally connected and locally path connected spaces;

CO 2 interpret product topology and quotient topology with examples;

CO 3 understand the Urysohn's lemma, Tietz's extension theorem, Urysohn's metrization theorem, Tychnoff theorem, one-point compactification and their applications;

CO 4 describe the basic properties of fundamental groups, covering spaces and their illustrations in determining the fundamental group of the circle.

Detailed Syllabus

- **Module 1:** Topological spaces, subspace topology, open and closed sets, neighbourhoods, limit points, interior and closure of a set, dense sets, base and subbase.
- **Module 2:** Countability axioms, continuous maps and homeomorphisms.
- **Module 3:** Compactness and connectedness, components, path connectedness, locally compact spaces, locally connected spaces, product topology.
- **Module 4:** Separation axioms, regular, completely regular and normal spaces, Urysohn's lemma, Tietz's extension theorem, Urysohn's metrization theorem (statement only), Tychonoff theorem, one-point compactification.
- **Module 5:** Topology of pointwise convergence, topology of compact convergence, compact-open topology.
- **Module 6:** Quotient spaces with examples (like torus, G/H, Klein's bottle, projective spaces, wedge sum of topological spaces etc.), homotopy, deformation retract, strong deformation retract, contractible spaces.
- **Module 7:** Homotopic paths and fundamental group π_1, simply connected topological spaces.
- **Module 8:** Covering spaces with examples, path lifting property, homotopy lifting property, computation of $\pi_1(S_1)$, lifting criterion (statement only), deck transformations and properly discontinuous group actions, construction of Universal cover, Galois correspondence for covering spaces.

References

[1] G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill Education.

[2] M. A. Armstrong, Basic Topology, Springer.

[3] J. Dugundji, Topology, McGraw-Hill Inc., US.

[4] J. Munkres, Topology, A first course, Pearson.

[5] J. L. Kelley, General Topology, Springer.

[6] J. Munkres, Elements of Algebraic Topology, CRC Press.

[7] A. Hatcher, Algebraic Topology, Cambridge University Press.

[8] G. E. Bredon, Topology and Geometry, Springer.

[9] J. J. Rotman, Introduction to Algebraic Topology, Springer.

Advanced Ordinary Differential Equation

Semester : VII	Course Type : T
Course ID : MATH402C17	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand how differential equations arise in real life problems; determine the existence and uniqueness of various initial value problems;

CO 2 solve homogeneous and inhomogeneous linear differential equations using various methods; solve various boundary value problems constructing Green's functions; apply the Sturm-Liouvelli problem to determine the eigenvalues and eigenfunctions of various boundary value problems that appear in various branches of science;

CO 3 apply the power series method or the Frobenious method to solve various differential equations; construct and apply various special functions, e.g. Legendre, Bessel, hypergeometric, Hermite, etc;

CO 4 investigate the local stability of the critical points of various autonomous systems that appear in the real world.

Detailed Syllabus

- **Module 1:** First order ordinary differential equation, initial value problems, existence theorem, basic theorems, Arzela-Ascoli theorem, theorem on convergence of solution of initial value problems, Picard - Lindelof theorem, Peano's existence theorem and corollaries, maximal interval of existence.
- **Module 2:** Linear second order ordinary differential equation with variable coefficients: Recapitulation of the basic theory; separation theorem and comparison theorem with applications. Exact equations and self-adjoint operator. Boundary value problems and Lagrange identity. Boundary value problems and Green's functions; construction of Green's functions, properties and applications. Sturm-Liouville problems; eigenfunctions expansion, orthogonality of eigenfunctions, completeness of the eigenfunctions.
- **Module 3:** Special Functions: Recapitulation of singular points, points at infinity, series solution and Frobenius method. Hypergeometric equation and functions; confluent hypergeometric functions and properties with applications. Hermite polynomials. Bessel's functions of first and second kinds, normal form of the Bessel's equation, orthogonality of Bessel functions, Bessel-Fourier series. Legendre equation, Legendre functions, orthogonality of Legendre functions and Legendre series.
- **Module 4:** Basic introduction to autonomous systems, phase portraits, isoclines, critical points, stability of the critical points, Lyapunov function, linearization about a critical point.

References

[1] Lawrence Perko, Differential Equations and Dynamical Systems, Springer.

[2] G. F. Simmons, Differential Equations with applications and historical notes, CRC Press.

[3] A. C. King, J. Billingham and S. R. Otto, Differential Equations, Cambridge University Press.

[4] G. Birkhoff, G-C Rota, Ordinary Differential Equations, Wiley and Sons.

[5] Carmen Chicone, Introduction to Ordinary Differential Equations, Springer-New York.

[6] R. P. Agarwal and D. O'Regan, Introduction to Ordinary Differential Equations, Springer.

[7] E. A. Coddington and N. Levinson, Theory of Ordinary Differential Equations, McGraw Hill.

[8] A. Chakraborty, Elements of Ordinary Differential Equations and Special Functions, New Age India International.

Differential Geometry

Semester : VIII	Course Type : T
Course ID : MATH451C20	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 explain the geometry of regular surfaces in three dimensions and hypersurfaces in $(n+1)$ dimensions; understand the first and second fundamental forms of regular surfaces, Gauss map, Gaussian curvature of regular surfaces;

CO 2 interpret the fundamental concepts of differentiable manifolds; illustrate the vector fields, integral curves, differential forms and exterior derivatives;

CO 3 understand the basic concepts of Riemannian manifolds such as Riemannian metrics, isometries, Levi-Civita connection, geodesics, exponential maps, complete Riemannian manifolds; elucidate the Hopf-Rinow theorem;

CO 4 compute explicitly isometry groups, geodesics, and sectional curvatures of some model spaces like Poincaré upper half plane and its disc model, hyperbolic n-space, Euclidean n-space, n-sphere.

Detailed Syllabus

Differentiable Manifolds

- **Module 1:** Smooth surfaces in three dimensions: definitions and various examples. Tangent space and normal vectors. Vector fields on surfaces, orientation on a surface. Regular surfaces in $\mathbb{R}^3$, hypersurfaces in $\mathbb{R}^{n+1}$.
- **Module 2:** Manifolds and smooth structures: definition and examples. Smooth mappings between manifolds. Tangent vectors as derivations, tangent and cotangent spaces. Jacobian matrix of a smooth map. Tangent and cotangent bundles. Vector fields and 1-forms as sections. Integral curves, flow of a vector field. Lie brackets.
- **Module 3:** Submanifolds; regular and critical points of a smooth map, immersion, submersion and embeddings. Differential k-forms and exterior derivatives.

Riemannian Geometry

- **Module 4:** First fundamental form of a surface. Length of a curve and surface area. Isometries of surfaces. Second fundamental form of a surface and different notions of curvature. Gauss map, Weingarten map or the shape operator of a surface. Gaussian curvature of a surface.
- **Module 5:** Riemannian metrics and Riemannian manifolds. Length of smooth curves in a Riemannian manifold and induced metric space, isometries and local-isometries. Affine connections and covariant derivatives, parallel transport, Christoffel symbols, Levi-Civita connection.
- **Module 6:** Geodesics, exponential maps, normal neighborhood, geodesics minimizing distances locally, complete Riemannian manifolds, Hopf-Rinow theorem (only statement).

- **Module 7:** Some model spaces like Poincare upper half plane and the disc model of hyperbolic 2-plane $\mathbb{H}^2$, Euclidean n-space $\mathbb{R}^n$, n-sphere $\mathbb{S}^n$, explicit computations of geodesics and (sectional) curvatures in such spaces.

References

[1] F. W. Warner, Foundations of Differentiable Manifolds and Lie Groups, Springer.

[2] N. J. Hicks, Notes on Differential Geometry, Van Nostrand.

[3] J. L. Dupont, Differential Geometry, Aarhus Universitet Matematisk Institut, (https://data.math.au.dk/publications/ln/1993/imf-ln-1993-62.pdf).

[4] M. P. Do Carmo, Riemannian Geometry, Birkhäuser.

[5] Gallot, Hulin, Lafontaine, Riemannian Geometry, Universitext-Springer.

[6] J. M. Lee, Riemannian Manifolds An Introduction to Curvature, Springer.

[7] L. Tu, Differential Geometry, Springer

[8] S. Helgason, Differential Geometry, Lie Groups, and Symmetric Spaces, AMS.

Classical Mechanics

Semester : VIII	Course Type : T
Course ID : MATH452C21	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 implement the Newtonian mechanics in various problems related to mathematics, physics and engineering;

CO 2 understand the concepts of constrained motion; derive the Lagrange's equations of motion with the use of D'Alemdert's principle and the principle of least action; use the concept of Hamiltonian in mechanics; apply the canonical transformations and Liouville's theorem in various mechanical problems;

CO 3 understand the central force problem and its application to the planetary motion; signify conserved quantities using Noether's theorem in various mechanical problems;

CO 4 solve several dynamical problems using known mathematical packages.

Detailed Syllabus

- **Module 1:** Review of Newtonian mechanics for a single particle and a system of particles; simple illustrations of Newton's equation of motion.
- **Module 2:** Constraints and their classifications, degrees of freedom, generalized coordinates, D' Alembert's principle, Lagrange's equation of motion for a system of holonomic constraints using D' Alembert's principle (differentiable principle) and Hamilton's principle (integral principle); Applications of the Lagrangian formulation; conservation theorems; central force problem.
- **Module 3:** Hamilton's equations of motion; cyclic coordinates and their consequences, Routhian, canonical transformations, examples of canonical transformations; Poisson's brackets; Liouville's theorem; Hamilton Jacobi theory; action angle variables; small oscillations; Noether's theorem.
- **Module 4:** Visualization of several dynamical problems using python/mathematica/matlab/maple.

References

[1] H. Goldstein, Classical Mechanics, Addison-Wesley Publishing Company.

[2] N. C. Rana and P. S. Joag, Classical Mechanics, Tata McGraw-Hill Education.

[3] L. D. Landau and E. M. Lifshitz, Mechanics, Butterworth Heinemann.

[4] S. T. Thornton and J. B. Marion, Classical Dynamics of Particles and Systems, Belmont, CA : Brooks/Cole.

[5] E. T. Whittaker, A Treatise on the Analytical Dynamics of Particles and Rigid Bodies with an introduction to the problem of three bodies, Cambridge University Press.

[6] R.P. Feynman, R.B. Leighton and M. Sands, The Feynman Lectures on Physics, Addison-Wesley Publishing Company.

Research Methodology

Semester : VII	Course Type : S
Course ID : MATH442MC05	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 do literature survey;

CO 2 pose scientific research problems; identify research objectives and formulate research techniques;

CO 3 type research papers using Latex;

CO 4 use various mathematical tools for research.

Detailed Syllabus

- **Module 1:** Scientific research and literature Survey.
- **Module 2:** Formulation of a research problem.
- **Module 3:** Developing a research plan: research objectives, information to be obtained and techniques to be adopted for solving the problem.
- **Module 4:** Research writing and presentation: Introduction to Latex and Beamer, write-ups in latex and beamer/power point presentations.
- **Module 5:**Mathematical software: Introduction to Mathematica/Matlab/Sage for solving numerical and computational problems.

References

[1] C.R. Kothari and G. Garg : Research Methodology: Methods and Techniques, 3rd Edition, New Age International Publishers, New Delhi.

[2] K. Prathapan : Research Methodology for Scientific Research, IK International, New Delhi.

[3] L. Lamport : LaTeX, a Document Preparation System, 2nd Edition, Addison-Wesley.

[4] Nicholas J. Higham : Handbook of Writing for the Mathematical Sciences, 2nd Edition, SIAM.

[5] Donald E. Knuth, Tracy L. Larrabee, and Paul M. Roberts : Mathematical Writing, Mathematical Association of America.

[6] David F. Griffiths, Desmond J.Higham : Learning LATEX, SIAM, Philadelphia.

[7] S.R. Otto and J.P.Denier : An Introduction to Programming and Numerical Methods in MATLAB, Springer.

[8] C-K. Cheung, G. E. Keough, Robert H. Gross, Charles Landraitis : Getting Started with Mathematica, Third Edition, John Wiley and Sons.

[9] SageMath - an open source mathematics software system: https://www.sagemath.org.

Research and Publication Ethics

Semester : VIII	Course Type : S
Course ID : MATH492MC06	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the responsible code of conduct for scientific research and publication; be aware of the role of research publication ethics as well as publication misconducts;

CO 2 learn hands-on skills to identify research malpractices and predatory publications;

CO 3 familiarize with indexing and citation databases, open access publications and various research metrics;

CO 4 use anti-plagiarism identification tools.

Detailed Syllabus

- **Module 1:** PHILOSOPHY AND ETHICS
  - Introduction to philosophy: definition, nature and scope, concept, branches
  - Ethics: definition, moral philosophy, nature of moral judgments and reactions.
- **Module 2:** SCIENTIFIC CONDUCT
  - Ethics with respect to science and research
  - Intellectual honest and research integrity
  - Scientific misconducts: falsification, fabrication, and plagiarism (FFP)
  - Redundant publications: duplicate and overlapping publications, salami slicing
  - Selective reporting and misrepresentation of data.
- **Module 3:** PUBLICATION ETHICS
  - Publication ethics: definition, introduction and importance
  - Best practices/standards setting initiatives and guidelines: COPE, WAME, etc.
  - Conflicts of interest
  - Publication misconduct: definition, concept, problems that lead to unethical behavior and vice verse, types
  - Violation of publication ethics, authorship and contributor ship
  - Identification of publication misconduct, complaints and appeals
  - Predatory publishers and journals
- **Module 4:** OPEN ACCESS PUBLISHING
  - Open access publications and initiatives
  - Journal finder/ journal suggestion tools viz. JANE, Elsevier Journal , Springer Journal Suggested, etc.

  - Software tools: Use of plagiarism software like Turnitin, Urkund and other open source software tools.

- **Module 5:** DATABASES AND RESEARCH METRICS
  - Databases: Indexing databases, Citation databases: Web of Science, Scopus, etc.
  - Research Metrics: Impact Factor of journal as per journal citation report, SNIP, SJR, IPP, Cite Score, Metrics: h-index, g index, i10 index, altmetrics

References

[1] Alasdair MacIntyre : A Short History of Ethics, Macmillan Publishers.

[2] A. Bird : Philosophy of Science, Routledge.

[3] P. Chaddah : Ethics in Competitive Research: Do not get scooped; do not get plagiarized, ISBN: 9789387480865.

[4] National Academy of Sciences, National Academy of Engineering (US) and Institute of Medicine (US) Committee on Science, Engineering, and Public Policy : On Being a Scientist: A Guide to Responsible Conduct in Research, Third Edition, National Academies Press.

[5] Indian National Science Academy (INSA) : Ethics in Science Education, Research and Governance, ISBN : 978-81-939482-1-7. https://www.insaindia.res.in/pdf/Ethics_Book.pdf

[6] P. Oliver : The Student's Guide to Research Ethics, Open University Press.

Topics for Elective Papers

Advanced Algebra

Semester : VII	Course Type : T
Course ID : MATH403C18A1	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the semi-direct products in group theory and learn nilpotent, solvable groups;

CO 2 understand the geometry of inner product spaces and applications of spectral theory; interpret the bilinear forms, alternating forms, and their classifications;

CO 3 explain the fundamental concepts of modules over a commutative ring, free modules; apply classification theorem of finitely generated modules over a PID on finite abelian groups and linear transformations; understand fundamental concepts of algebraic number fields, infinite fields of positive characteristic;

CO 4 apply finite fields to solve various numerical problems.

Detailed Syllabus

- **Module 1:** Semi-direct products; composition series, exact sequences; solvable and nilpotent groups.
- **Module 2:** Inner-product spaces, Gram-Schmidt orthogonalisation, bi-linear forms, definition of unitary, hermitian, normal, real symmetric and orthogonal linear transformations, spectral theorems; multi-linear forms, alternating forms.
- **Module 3:** Modules over commutative rings, examples: vector spaces, commutative rings, $\mathbb{Z}$ modules, $F[X]$-modules; submodules. Quotient modules, homomorphisms, isomorphism theorems, $Hom_R(M, N)$ for R-modules M, N, generators and relations for modules, direct products and direct sums, direct summands, free modules, finitely generated modules.
- **Module 4:** Field Theory: Field extensions, finite and algebraic extensions, algebraic closure, splitting fields, normal extensions, separable, inseparable and purely inseparable extensions, ruler and compass constructions. Finite fields: Structure of finite fields, existence and uniqueness theorems; examples of construction of finite fields of order p^2, p^3 etc., primitive elements, minimal polynomials of elements, irreducible and primitive polynomials.

References

[1] D. S. Dummit and R. M. Foote, Abstract Algebra, Wiley.

[2] S. Lang, Algbera, Springer.

[3] T. W. Hungerford, Algbera, Springer.

[4] K. Hoffman and R. Kunze, Linear Algebra, Prentice-Hall, Inc.

[5] M. R. Adhikari and A. Adhikari, Basic Modern Algebra with Applications, Springer.

Advanced Complex Analysis

Semester : VII	Course Type : T
Course ID : MATH403C18A2	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 identify several advanced topics in Complex Analysis e.g. the idea of a Riemann surface of a function; conformal mappings, univalent functions etc;

CO 2 understand Hurwitz's theorem, Normality, Montel's theorem, Riemann mapping theorem, Schwarz-Christoffel formula; understand Marty's theorem, Zalcman's lemma, and Fundamental normality; explain the proofs of the little and the great Picard's theorem;

CO 3 explain the factorization theorems due to Weierstrass and Hadamard with the order and genus of an entire function; apply Runge's theorem and its consequences;

CO 4 conceptulize the notion of analytic continuation, Schwarz reflection principle, Monodromy theorem; understand Bloch's theorem on the range of an analytic function; have the ideas of harmonic, subharmonic and superharmonic functions.

Detailed Syllabus

- **Module 1:** Conformal mappings, level curves, survey of elementary mappings, elementary Riemann surfaces.
- **Module 2:** Revision of compactness and convergence in the space of analytic functions, convergence on compact subsets, Hurwitz's classical version, normality, Montel's theorem, Riemann mapping theorem, Schwarz-Christoffel formula.
- **Module 3:** Weierstrass spherical convergence theorem, spherical metric, spherical derivative, Marty's theorem, Zalcman's lemma, Bloch's principle, fundamental normality.
- **Module 4:** Weierstrass factorization theorem, factorization of the Sine function, Gamma function, Riemann Zeta function, Jensen's formula, genus and order of an entire function, Hadamard factorization theorem.
- **Module 5:** Runge's theorem, simple connectedness, Mittag-Leffler's theorem.
- **Module 6:** Analytic continuation and Riemann surfaces, Schwarz reflection principle, analytic continuation along a path, Monodromy theorem, Sheaf of germs of analytic functions on an open set, analytic manifolds, covering spaces.
- **Module 7:** Basic properties of harmonic functions, harmonic functions on a disk, subharmonic and superharmonic functions, Dirichlet problem, Green's functions, harmonic measure.
- **Module 8:** Bloch's Theorem, the little and the great Picard's theorem.

References

[1] J. B. Conway, Functions of One Complex Variable, Narosa Publishing House.

[2] E. M. Stein and R. Shakarchi, Complex Analysis, Princeton University Press.

[3] L. V. Ahlfors, Complex Analysis, McGraw-Hill Education.

[4] T. W. Gamelin, Complex Analysis, Springer.

[5] W. Rudin, Real and Complex Analysis, McGraw-Hill Education.

[6] S. G. Krantz, Complex Analysis: The Geometric Viewpoint, The Mathematical Association of America.

Number Theory and Cryptography

Semester : VII	Course Type : T
Course ID : MATH403C18A3	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 apply number theory in real life problems; understand the fundamentals of number theory and its application to cryptography;

CO 2 understand network security threats, security services, and countermeasures; apply cryptography to real life problems;

CO 3 implement public key cryptography using python or sage programming language;

CO 4 implement probabilistic primality testing algorithm using python or sage programming language.

Detailed Syllabus

- **Module 1:** (Number Theory) Divisibility, primes and unique factorisation; GCD and Euclidean algorithm and its extension for computing multiplicative inverses; arithmetic functions or number theoretic functions: sum and number of divisors, (totally) multiplicative functions, the greatest integer function, Euler's phi-function, Möbius function; definition and properties of the Dirichlet product; some properties of the Euler's phi-function, statement of the prime number theorem. Linear Diophantine equations, congruences and complete residue systems; quadratic residues, quadratic reciprocity and the law of quadratic reciprocity, Euler's criterion, Legendre symbol and Jacobi symbol, Euler-Fermat theorem, Wilson's theorem, Chinese remainder theorem.
- **Module 2:** (Cryptography) Public-key encryption, Solovay–Strassen primality testing algorithm, notion of algorithms and their complexity, order notation, polynomial time algorithm, idea of hardness of factoring and discrete logarithm problem; basics of Diffie-Hellman key agreement and RSA encryption and decryption.
- **Module 3:** Symmetric or Private Key Cryptography: Private key encryption, perfectly secure encryption and its limitations, one time Pad, semantic security.
- **Module 4:** Stream Cipher: Boolean function, LFSR, pseudo-random number generator, nonlinear combiner model, linear complexity, Walsh transformation, Hadamard matrix, Correlation immunity, attacks on Boolean functions, S-Box, some stream ciphers such as RC4, Attack on RC4.
- **Module 5:** Block Cipher: Data Encryption Standard (DES), modes of operations, the Advanced Encryption Standard (AES), basic algorithms.
- **Module 6:** Cryptanalysis of Stream and Block ciphers, Linear and Differential Attacks.
- **Module 7:** Hash functions: Security properties of Hash functions, birthday attack, MAC, Construction of Hash functions, Merkle-Damgard construction. Exposure to SHA256.
- **Module 8:** Complexity analysis of algorithms: time complexity, space complexity, polynomial time algorithm, exponential algorithm, probabilistic algorithm, P, NP.

- **Module 9:** PKCs and Signature Schemes: Goldwasser-Micali, Paillier. Elliptic curves: properties of elliptic curves, elliptic curve over real and modulo a prime, torsion points, Public Key Infrastructure (PKI), Exposure to Signature Schemes.
- **Module 10:** Secret Sharing Schemes: Shamir's Secret Sharing Scheme, more on Secret Sharing Schemes such as cheating immune, cheating identifiable etc, visual cryptography, DNA secret sharing scheme.
- **Module 11:** Sage/Python implementation of various primitives for cryptographic schemes.

References

[1] M. R. Adhikari and A. Adhikari, Basic Modern Algebra with Applications, Springer.

[2] Steven D. Galbraith : Mathematics of Public Key Cryptography, Cambridge university press.

[3] D. R. Stinson, Cryptography : Theory & Practice, CRC Press Company.

[4] Jeffery Hoffstein, Jill Pipher, J.H.Silverman : An Introduction to Mathematical Cryptography, Springer.

[5] Jonathan Katz, Yehuda Lindell : Introduction to Modern Cryptography, Chapman & Hall/CRC.

[6] Neal Koblitz : A course in number theory and cryptography, Springer-Verlag, 2nd edition.

[7] D. M. Burton : Elementary Number Theory, Wm. C. Brown Publishers, Dulreque, Lowa.

[8] Kenneth. H. Rosen : Elementary Number Theory & Its Applications, AT&T Bell Laboratories, Addition-Wesley Publishing Company, 3rd Edition.

[9] Kenneth Ireland & Michael Rosen : A Classical Introduction to Modern Number Theory, 2nd edition, Springer-Verlag.

[10] Richard A Mollin : Advanced Number Theory with Applications, CRC Press, A Chapman & Hall Book.

Operations Research

Semester : VII	Course Type : T
Course ID : MATH403C18B1	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 efficiently handle invenotry models and implement it in practical problems; check the robustness of the developed models and make appropriate decisions;

CO 2 construct and solve real world problems with queuing theory;

CO 3 develop and analyse multi-objective and multi-stage programming, goal programming and dynamic programming, quadratic programming;

CO 4 apply learned techniques in agricultural planning, biotechnology, data analysis, distribution of goods and resources, emergency and rescue operations, engineering etc.

Detailed Syllabus

- **Module 1:** Deterministic Inventory control Models: Nature of inventory problems. Structure of inventory systems. Definition of inventory problem. Important parameters associated with inventory problems. Variables in inventory problems. Controlled and uncontrolled variables. Types of inventory systems and inventory policies. Deterministic inventory models / systems. Harris-Wilson model. Economic lot size systems. Sensitivity of the lot size systems. Order level systems and their sensitivity analysis. Order level lot size and their sensitivity studies.
- **Module 2:** Probabilistic Inventory control Models: Probabilistic demand models. Expected cost. Probabilistic order level systems. Probabilistic order level systems with instantaneous demand. Probabilistic order level systems with uniform demand. Probabilistic order level systems with lead time. Discrete and continuous probability versions of the models. Problems on the two versions of the models. Newspaper boy problem.
- **Module 3:** Queuing Theory: Definitions – queue (waiting line), waiting costs, characteristics (arrival, queue, service discipline) of queuing system, queue types (channel vs. phase), Kendall's notation, Little's law, steady state behaviour, Poisson's Process & queue, models with examples - M/M/1 and its performance measures; M/M/C and its performance measures; brief about some special models (M/G/1).
- **Module 4:** Introduction to multi-objective and multi-stage programming, goal programming and dynamic programming, quadratic programming.
- **Module 5:** Introduction and basic differences between PERT and CPM, steps of PERT/CPM Techniques, PERT/CPM network components and precedence relationships, Fulkerson's 'i-j' rule, critical path analysis, forward and backward pass methods, floats of an activity, project costs by CPM, probability in PERT analysis, project crashing, time cost trade-off procedure, updating of the project, resource allocation.

References

[1] John A. Muckstadt, Amar Sapra, Principles of Inventory Management, Springer. Sven Axsater, Inventory Control, Springer.

[2] E. Nadder, Inventory Systems, John Wiley and Sons.

[3] G. Hadley and T. M. Whitin, Analysis of Inventory Systems, Prentice Hall.

[4] R. J. Tersine and M. Hays, Principles of Inventory and Material Management, Pearson.

[5] A. Ravindran, Don T. Phillips, James J. Solberg, Operations Research: Principles and Practice, Wiley.

[6] H. S. Taha, Operations Research, Pearson Education.

[7] F.S. Hillier, G.J. Lieberman, Introduction to Operations Research, McGraw Hill Education.

[8] S. Maurice, A. Yaspan, L. Friedman, OR methods and Problems, Wiley.

[9] S. D. Sharma, Operations Research, Kedar Nath.

Tensor Analysis and Integral Transforms

Semester : VII	Course Type : T
Course ID : MATH403C18B2	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 do computations using tensors in both coordinate free way and in coordinates; apply tensors in manifolds and higher dimensional geometry and in mechanics;

CO 2 develop and apply Fourier transform in solving Partial differential equations and initial value problems;

CO 3 develop Laplace transform of functions; apply Laplace transform and its several properties for solving partial differential equations and ordinary differential equations; develop Hankel transform of functions; apply Hankel transform to solve partial differential equations; develop Mellin transform of functions;

CO 4 acquaint with the applications of integral transform in problems of mechanics, fluid dynamics, elasticity etc.

Detailed Syllabus

- **Module 1:** Introduction to tensors, tensor algebra, tensor calculus.
- **Module 2:** Fourier Transform, Fourier integral theorem, Riemann-Lebesgue lemma, cosine and sine transforms, inversion theorem, properties of Fourier transformation with applications, derivatives, convolution theorem, convolution of Fourier sine/cosine transform. Application of Fourier transform in ordinary and partial differential equations.
- **Module 3:** Laplace Transform, functions of exponential order and existence condition for Laplace transform. Properties of Laplace transform with applications, inversion of Laplace transform, application in solving ordinary and partial differential equations.
- **Module 4:** Introduction to Mellin and Hankel transforms, their properties and applications.

References

[1] B. Spain, Tensor Calculus, a concise course, Dover Publications, Inc.

[2] L. Brand, Vector and Tensor Analysis, John Wiley & Sons.

[3] H. Lass, Vector and Tensor Analysis, McGraw-Hill Book Company, Inc.

[4] I. N. Sneddon, Use of Integral Transforms, McGraw Hill.

[5] H. G. ter Morsche, J. C. van den Berg, E. M. van de Vrie, Fourier and Laplace Transforms, Cambridge University Press.

[6] I. N. Sneddon, Fourier Transform, Dover Publications.

[7] R. N. Bracewell, Fourier Transform and its Applications, McGraw Hill.

[8] J. L. Schiff, Laplace Transform Theory and Applications, Springer.

Mechanics

Semester : VII	Course Type : T
Course ID : MATH403C18B3	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the concept of equilibrium phase under the action of various forces; understand the concept of the moment of a force about a point or line, and couple and their applications in problems; calculate the center of gravity of different objects; know how friction plays an important thing in statics and dynamics;

CO 2 understand the dynamics of a particle moving in one, two and three dimensions;

CO 3 understand the concept of a central force field and the dynamics of a particle under the central force field, and Kepler's laws of planetary motion;

CO 4 understand the concepts of moment of inertia and product of inertia associated with the objects.

Detailed Syllabus

- **Module 1:** Coplanar forces, parallel forces, moments, couples, astatic equilibrium, friction, principle of virtual work, center of gravity for different bodies, stable and unstable equilibrium, forces in three dimensions, general conditions of equilibrium.
- **Module 2:** Newtonian mechanics for a single particle; rectilinear motion for constant and variable accelerations; motion in a resisting medium; simple harmonic motion, disturbed simple harmonic motion; conservation theorems
- **Module 3:** Motion in a plane; dynamics of a particle in a central force field, central orbits; planetary motion and Kepler's laws.
- **Module 4:** Moments and products of inertia of rigid bodies; parallel axis theorem, perpendicular axis theorem; Euler equations.

References

[1] S. L. Loney, Elements of Statics and Dynamics 1 and 2, Arihant Publications.

[2] S. L. Loney, An Elementary treatise on Dynamics of particle and rigid bodies, New Age International Private Limited.

[3] D. Kleppner and R. Kolenkow, An Introduction to Mechanics, Cambridge University Press.

[4] F. Chorlton, Textbook of Dynamics, John Wiley & Sons.

[5] J. L. Synge, B. A. Griffith, Principles of Mechanics, Mcgraw Hill

[6] A. S. Ramsey, Statics, Cambridge University Press.

Measure Theory

Semester : VIII	Course Type : T
Course ID : MATH453C22A1	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the σ-algebra, Abstract Measure space, different extension theorems and completion of a measure space;

CO 2 understand Lebesgue measure, Lebesgue measurable sets, Lebesgue measurable functions and Lebesgue integration;

CO 3 understand measures and integrals on product spaces, signed measures, the Radon-Nikodym theorem and its applications;

CO 4 understand the properties of L_p-spaces and the fundamental theorem for Lebesgue integral.

Detailed Syllabus

- **Module 1:** Algebra, σ-algebra, monotone class theorem, measure spaces.
- **Module 2:** Outer measures, Caratheodory extension theorem, pre-measures, Hahn-Kolmogorov extension theorem, uniqueness of the extension, completion of a measure space.
- **Module 3:** Lebesgue measure and its properties.
- **Module 4:** Measurable functions and their properties, modes of convergence.
- **Module 5:** Integration, monotone convergence theorem, Fatou's Lemma, dominated convergence theorem.
- **Module 6:** Product measures, Fubini's theorem.
- **Module 7:** L_p-spaces, Riesz Representation theorem.
- **Module 8:** Signed measure, Radon-Nikodym theorem and its applications.
- **Module 9:** Fundamental theorem of calculus for Lebesgue integrals.

References

[1] T. Tao, An Introduction to Measure Theory, American Mathematical Society.

[2] I. K. Rana, An Introduction to Measure and Integration, Narosa.

[3] P. R. Halmos, Measure Theory, Springer.

[4] H. L. Royden, Real Analysis, Pearson.

[5] W. Rudin, Real and Complex Analysis, McGraw Hill Education.

Lie Algebra and Representation Theory

Semester : VIII	Course Type : T
Course ID : MATH453C22A2	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand Lie algebras with examples, their fundamental properties; solvable and nilpotent Lie algebras, theorems of Lie and Engel;

CO 2 explain the fundamental concepts of semisimple and simple Lie algebras such as Cartan subalgebras, root space decompositions, Dynkin diagrams; understand the classification of complex simple Lie algebras;

CO 3 explain the representation theory of $\mathfrak{sl}_2(\mathbb{C})$ and for general complex semisimple Lie algebras, theory of weights;

CO 4 understand classification of irreducible representations of complex semisimple Lie algebras in terms of weights, Weyl's character formula.

Detailed Syllabus

- **Module 1:** Lie algebra and examples, solvable and nilpotent Lie algebra, theorems of Lie and Engel, Killing form, semisimple and simple Lie algebra, reductive Lie algebra, compact Lie algebra.
- **Module 2:** Cartan subalgebra of a semisimple Lie algebra, root space decomposition of a complex semisimple Lie algebra, Chevalley basis and structural constants, real forms and compact real forms of a complex semisimple Lie algebra. classical complex simple Lie algebras.
- **Module 3:** Concepts of system of positive roots and simple roots, Dynkin diagram, exceptional complex simple Lie algebras, classification of all complex simple Lie algebras(statement only).
- **Module 4:** Representations of a reductive Lie algebra and examples, irreducible representations, the adjoint representation, representations of $\mathfrak{sl}(2, \mathbb{C})$, weights, maximal vectors, Verma module.
- **Module 5:** Finite dimensional representations of a complex semisimple Lie algebra, integral weights, dominant integral weights, classification of irreducible representations of a complex semisimple Lie algebra.
- **Module 6:** Casimir element, characters, Kostant's multiplicity formula, Weyl's formula.

References

[1] J.E. Humphreys, Introduction to Lie Algebras and Representation Theory, Springer.

[2] S. Helgason, Differential Geometry, Lie Groups, and Symmetric Spaces, AMS.

[3] W. Fulton, J. Harris, Representation Theory: A First Course, Springer.

[4] N. Jacobson, Lie Algebras, Dover.

Geometric Group Theory

Semester : VIII	Course Type : T
Course ID : MATH453C22A3	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 explain free groups and understand their basic properties; construct a group from its presentation, and understanding of the word, conjugacy and isomorphism problems on finitely-presented groups, Dehn's theorem;

CO 2 explain the Cayley graphs of finitely generated groups and associated word metrics; prove the Svarc-Milnor Lemma and apply it;

CO 3 understand the growth of groups, examples of groups of different growth types: polynomial, intermediate and exponential growth, statement of Gromov's theorem on polynomial growth;

CO 4 understand the delta-hyperbolic metric spaces, hyperbolic groups and boundaries at infinity.

Detailed Syllabus

- **Module 1:** Recap - Fundamental groups. Covering spaces and the universal covering space. Deck transformations - π_1 action on the universal cover. Seifert-van Kampen theorem.
- **Module 2:** Free groups. Presentation of groups: generators and relations. Finitely presented groups. Direct and semi-direct products. Group extensions. Free product, amalgamated free product and HNN-extension of groups, normal forms of elements in these products. Examples.
- **Module 3:** Cayley graphs of free groups. Cayley graphs of finitely generated groups and the word metric. Group action on spaces, orbits and stabilizers. Group actions on Cayley graphs as metric spaces. Free groups and action on trees, Nielsen-Schreier theorem, ping-pong lemma.
- **Module 4:** Quasi-isometry (QI): examples. QI types of groups, QI invariants. Quasi-geodesics. Švarc–Milnor Lemma: commensurability and other applications. Uniform lattices and QI.
- **Module 5:** Growth function and growth types of a finitely generated group. Groups of polynomial growth and (virtual) nilpotence. Groups of intermediate and exponential growth. Growth series.
- **Module 6:** (Gromov) Hyperbolic metric spaces. QI-invariance of (delta-)hyperbolicity. Hyperbolic groups, elements of infinite order, centralisers, quasi-convex subgroups. Dehn's algorithm and the word problems in hyperbolic groups. Growth function of a hyperbolic group. (If time permits) Ends and boundaries at infinity.

References

[1] C. Löh, Geometric Group Theory, Universitext, Springer.

[2] M. Clay, D. Margalit, Office Hours, with a Geometric Group Theorist, Princeton UP.

[3] O. Bogopolski, Introduction to Group Theory, Hindustan Book Agency.

[4] B. Bowditch, A course on Geometric Group Theory, MSJ Memoirs.

[5] C. Drutu, M. Kapovich, Geometric Group Theory, Coll. Pub. Vol. 63, AMS.

[6] J.P. Serre, Trees, Springer Monographs in Mathematics - Springer.

Mathematical Modelling

Semester : VIII	Course Type : T
Course ID : MATH453C22B1	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 mathematically model real life problems;

CO 2 understand and apply discrete and continuous models in physical and biological systems;

CO 3 understand the autonomous system and apply the bifurcation analysis;

CO 4 solve modeling problems and doing sensitivity analysis using programming softwares.

Detailed Syllabus

- **Module 1:** Introduction: an overview of mathematical modelling
- **Module 2:** Discrete models: Motivation, examples, solution and equilibrium of the discrete models, Cobwebbding method, general theory and analytical methods.
- **Module 3:** Continuous models: Motivation and derivation of continuous models, Differential Equation models, Separation of variables, linear equations.
- **Module 4:** Sensitivity Analysis: motivation and application.
- **Module 5:** Systems of difference equations (discrete): Analytical methods and examples.
- **Module 6:** Systems of Differential equations (Continuous): Motivations and some examples, Nondimensionalization, analytical methods, higher-order systems.
- **Module 7:** Bifurcation analysis: Saddle-node, Transcritical, Pitchfork (both one and two dimensions), introduction of Hopf bifurcation, normal form of Hopf bifurcation.
- **Module 7:** Programming using any mathematical software
  - Plot for discrete models
  - Plot for continuous models
  - Plot for sensitivity
  - Plot for bifurcation

References

[1] F. R. Giordano, M. D. Weir and W. P. Box, A first course in mathematical modelling, Thomson Learning.

[2] L. E. Keshet, Mathematical models in biology, SIAM.

[3] J. D. Murray, Mathematical Biology, Springer.

[4] F. Brauer, P.V.D.Driessche, J. Wu, Mathematical Epidemiology, Springer.

Special Theory of Relativity

Semester : VIII	Course Type : T
Course ID : MATH453C22B2	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the limitations of the Newtonian mechanics;

CO 2 understand the Lorentz transformations and Lorentz group;

CO 3 understand the concept of length contraction and time dilation; understand the 4-dimensional Minkowskian space-time and its consequences;

CO 4 understand four vectors, relativistic mechanics, equivalence of mass and energy (the famous equation $E = mc^2$).

Detailed Syllabus

- **Module 1:** Newton's laws and inertial frames, Galilean transformations, Newtonian relativity, the Michelson-Morley experiment, Einstein's thoughts and his postulates of special theory of relativity.
- **Module 2:** The relativity of simultaneity, Lorentz transformations; mathematical properties of Lorentz transformations, spacetime invariant, length contraction, time dilation, twin paradox, relativistic addition of velocities.
- **Module 3:** Minkowski's spacetime, space-like, time-like and light-like intervals, lightcone; four vectors, geometry of four vectors, proper time, relativistic mass, momentum and energy, equivalence of mass and energy, energy-momentum tensor.

References

[1] R. Resnick, Introduction to Special Relativity, John Wiley & Sons.

[2] A. P. French, Special Relativity, CRC Press.

[3] S. Banerjee and A. Banerjee, The Special Theory of Relativity, PHI.

[4] Ray D'Inverno, Introducing Einstein's Relativity, Clarendon Press.

[5] W. Rindler, Relativity - Special, general and cosmological, Oxford University Press

Qualitative Theory of ODE

Semester : VIII	Course Type : T
Course ID : MATH453C22B3	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the existence and uniqueness of the solutions of the ordinary differential equations; have concepts on conjugacy and equivalency of flows;

CO 2 investigate the qualitative behavior of linear flows on R^n; classify linear flows on R^2 up to C^0-equivalency;

CO 3 characterize the qualitative nature of a flow in a neighborhood of a singularity;

CO 4 understand the local structure of limit sets.

Detailed Syllabus

- **Module 1:** Fundamental theorem for existence and uniqueness of solutions for ordinary differential equations, Gronwall's inequality, dependence on initial conditions and parameters, maximal interval of existence, global existence of solutions, vector fields and flows, topological conjugacy and equivalency.
- **Module 2:** Linear flows on $\mathbb{R}^n$: Linear first order autonomous systems, the matrix exponential, Jordan canonical forms, invariant subspaces, stability theory, classification of linear flows, fundamental matrix solution, non-homogeneous linear systems, periodic linear systems and Floquet theory.
- **Module 3:** Nonlinear Systems: Local analysis, the local stable manifold theorem, the Hartman-Grobman theorem, Nonhyperbolic singularities in $\mathbb{R}^2$, the center manifold theorem, stability and Lyapunov function, normal form theory, gradient and Hamiltonian systems.
- **Module 4:** Nonlinear Systems: Global Analysis α and ω limit sets of an orbit, attractors, periodic orbits and limit cycles, separatrix cycles, the Poincaré map, characteristic exponents and characteristic multipliers, the stable manifold theorem for periodic orbits, the Poincaré-Bendixson theorem in $\mathbb{R}^2$, Bendixson and Dulac's criterion, Liénard systems.

References

[1] C. Chicone: Ordinary differential Equations with applications, Springer.

[2] L.D. Perko: Differential Equations and Dynamical Systems, Springer.

[3] E. A. Coddington and N. Levinson: Theory of Ordinary Differential Equations, Tata McGraw-Hill.

[4] M. W. Hirsch, S. Smale & R. L. Devaney: Differential Equations, Dynamical Systems. and an Introduction to Chaos, Academic Press, Elsevier.

[5] S. Wiggins: Introduction to Applied Nonlinear Dynamical System and Chaos, Springer.

[6] C. Robinson: Dynamical Systems: Stability, Symbolic Dynamics and Chaos, CRC Press.

Differential Calculus

Semester : I	Course Type : T
Course ID : MATH104MC01	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 acquaint with the real numbers and learn the construction of number systems; understand the concept of sequence and series of real numbers and their convergences;

CO 2 understand the limit and continuity of a function at a point; familiarize with the important properties of the continuous functions on closed intervals;

CO 3 acquire knowledge of derivatives of functions at points with various examples and its geometrical and physical interpretation;

CO 4 apply the important results of differentiable functions on intervals and their applications to differential calculus, e.g. maxima-minima, tangent, normals, etc; apply indeterminate forms and use of L'Hôpital's rule.

Detailed Syllabus

Module 1: Real Numbers: Axiomatic definition, Archimedean property, limit supremum, limit infimum.

Module 2: Sequence of real numbers: convergence, Cauchy criteria and other elementary properties. Series of real numbers, absolute and conditional convergence of series.

Module 3: Real-valued functions defined on an interval: Limit of a function (Cauchy's definition). Algebra of limits. Continuity of a function at a point and in an interval. Acquaintance with the important properties of continuous functions on closed intervals.

Module 4: Derivative its geometrical and physical interpretation. Sign of derivative, Monotonic increasing and decreasing functions. Relation between continuity and differentiability.

Module 5: Successive derivative (Leibnitz's Theorem and its application).

Module 6: Rolle's theorem; Mean Value Theorems and expansion of functions like e^x; $\sin x$; $\cos x$; $(1+x)^n$; $\ln(1+x)$ (with validity of regions).

Module 7: Applications of differential calculus: Maxima and minima, tangents and normals.

Module 8: Indeterminate Forms: L'Hôspital's Rule.

Books Recommended:

1. R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, Inc.
2. T. M. Apostol, Calculus (Vol. I), Wiley.
3. D. V. Widder, Advanced Calculus, Dover Publications.
4. S. Narayan, Differential Calculus, S. Chand.

Integral Calculus and Differential Equations

Semester : II	Course Type : T
Course ID : MATH154MC02	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 integrate rational functions and apply reduction formulas of integrations; compute definite integrals as the area under a curve;

CO 2 understand the concept of Riemann integral to functions defined on unbounded intervals or to unbounded functions i.e. the concept of improper Riemann integrals, related tests and Beta and Gamma functions;

CO 3 apply integral calculus to find the length of a curve, quadrature, volume and surface areas of solids formed by revolution of plane curve and areas, etc;

CO 4 understand the genesis of ordinary differential equations; apply various techniques for solving first and higher order linear differential equations.

Detailed Syllabus

Module 1: Integration of the form $\int \frac{dx}{a+b\cos x}$, $\int \frac{l\sin x+p\cos x}{m\sin x+n\cos x}dx$ and integration of rational functions. Reduction formulae of $\int \sin^m x\cos^n xdx$; $\int \tan^n xdx$ and $\int \frac{\sin^m x}{\cos^n x}dx$ and associated problems (m and n are non-negative integers).

Module 2: Evaluation of definite integrals. Preliminaries of Riemann integration. Integration as the limit of a sum.

Module 3: Definition of Improper Integrals: Statements of (i) μ-test, (ii) Comparison test. Use of Beta and Gamma functions.

Module 4: (Applications of integral calculus) rectification, quadrature, finding c.g. of regular objects, volume and surface areas of solids formed by revolution of plane curve and areas.

Module 5: Introduction to differential equations; order and solution of an ordinary differential equation (ODE) in presence of arbitrary constants; formation of ODE.

Module 6: First order differential equations: (i) Variables separable, (ii) Homogeneous equations and equations reducible to homogeneous forms, (iii) Exact equations and those reducible to such equations, (iv) Euler's and Bernoulli's equations (Linear), (v) Clairaut's equations: General and singular solutions; orthogonal trajectories.

Module 7: Second order linear equations: Second order linear differential equations with constant coefficients. Euler's Homogeneous equations.

Books Recommended

1. S. Narayan: Integral Calculus, S. Chand.

2. T. M. Apostol: Calculus (Vol. I), Wiley.
3. S. L. Ross: Differential Equations, John Wiley and Sons.
4. G. F. Simmons: Differential Equation with Applications and Historical Notes, CRC Press.

Algebra I

Semester : III	Course Type : T
Course ID : MATH205MC03	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 use De Moivre's theorem in a number of applications to solve mathematical problems;

CO 2 determine roots of real and complex polynomials using various methods;

CO 3 get a preliminary idea about groups.

CO 4 applications of rings and fields.

Detailed Syllabus

Module 1: (Complex Numbers) De Moivre's theorem and its applications. Exponential, sine, cosine and logarithm of a complex number. Definition of e^z, inverse circular and hyperbolic functions.

Module 2: (Theory of Equations) Fundamental theorem of algebra. Polynomials with real coefficients: Descarte's Rule of sign and its applications. Relation between roots and coefficients. Symmetric functions of roots, Transformations of equations. Solution of a cubic and biquadratic.

Module 3: (Introduction to Group Theory) Definition and examples, cyclic group, symmetric group, alternating group. Elementary properties of groups. Order of an element in the group, subgroup, quotient group, normal subgroup, homomorphism and isomorphism.

Module 4: (Rings and Integral Domains) Definition and examples. Subrings and ideals. Quotient rings. Homomorphism and isomorphism of rings.

Module 5: (Fields) Definition and examples, relation with integral domains.

Books Recommended

1. S. K. Mapa, Classical Algebra, Levant.
2. J. B. Fraleigh, First Course in Abstract Algebra, Narosa.
3. M. K. Sen, S. Ghosh and P. Mukhopadhyay, Topics in Abstract Algebra, University Press.

Algebra II

Semester : IV	Course Type : T
Course ID : MATH255MC04	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 have basic understanding of vector spaces;

CO 2 recognize consistent and inconsistent systems of linear equations by the row echelon form of the augmented matrix, using rank; and solve consistent systems of linear equations;

CO 3 have basic knowledge of linear transformations;

CO 4 find eigenvalues and corresponding eigenvectors for a square matrix.

Detailed Syllabus

Module 1: Vector (Linear) space over a field. Subspaces. Linear combinations. Linear dependence and independence of a set of vectors. Linear span. Basis. Dimension. Replacement theorem. Extension theorem. Deletion theorem.

Module 2: Row space and column space of a matrix. Determinant and trace of a matrix. Rank of a matrix. Rank$(AB) \leq$ min(Rank A; Rank B).

Module 3: System of linear homogeneous equations: Solution space of a homogeneous system and its dimension. System of linear non-homogeneous equations: Necessary and sufficient condition for the consistency of the system. Method of solution of the system of equations.

Module 4: Linear transformation (L.T.) on vector Spaces: Null space. Range space. Rank and Nullity, Sylvester's law of Nullity. Inverse of linear transformation. Non-singular linear transformation. Change of basis by linear transformation. Vector spaces of linear transformation.

Module 5: Characteristic equation of a square matrix. Eigen-value and eigen-vector. Invariant subspace. Cayley-Hamilton theorem. Simple properties of eigen value and eigen vector, diagonalization.

Books Recommended

1. S. Kumaresan, Linear Algebra: A Geometric Approach, PHI.
2. B. Rao, Linear Algebra, HBA (TRIM).
3. S. H. Friedberg, A. Insel and L. E. Spence, Linear Algebra, Pearson.

Joy of Numbers 1

Semester : I	Course Type : S
Course ID : MATH141MDC01	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 describe a set using set notation and terminology; demonstrate set relations and operations by Venn diagrams;

CO 2 give an idea of the cardinality of a set and Cantor's theory of countability;

CO 3 illustrate an understanding of the Principle of Mathematical Induction including examples;

CO 4 describe different series of numbers with specific reference to the Fibonacci series, golden ratio along with its applications in nature.

Detailed Syllabus

Module 1: A brief historical outline of modern mathematics.

Module 2: Basic set theory [Cardinality of a set, Power set, Venn diagrams] .

Module 3: Elementary mumber theory [Prime Numbers].

Module 4: Different series of numbers [special reference to Fibonacci series. Golden ratio, its presence in arts and nature].

Books Recommended

1. T.A. Garrity: All the Math You Missed: (But Need to Know for Graduate School).
2. Bell: Men of Mathematics
3. Polya: How to solve it
4. Ribenboim: The Little Book of Bigger Primes

Joy of Numbers 2

Semester : II	Course Type : S
Course ID : MATH191MDC02	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 find area of closed curves;

CO 2 application of Pythagorus' theorem;

CO 3 understand the concept of mean;

CO 4 understand the concept of irrational numbers and their approximation.

Detailed Syllabus

Module 1: Areas of closed figures.

Module 2: Irrational numbers and Pythagorean principle applied to a square.

Module 3: Geometric and arithmetic mean.

Module 4: Approximation of irrational numbers (How do we do it?)

Books Recommended

1. H. Rademacher and O. Toeplitz, The enjoyment of Mathematics: Selections from Mathematics for the amateur.
2. Bell, Men of Mathematics

Elementary geometry: The conic sections

Semester : II	Course Type : S
Course ID : MATH192MDC03	Full Marks : 100

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the geometry of change of coordinates by translation and rotation in two dimension;

CO 2 visualization of the above concepts.

CO 3 identify conics represented by general second degree equations in two variables.

CO 4 compute and trace conics.

Detailed Syllabus

Module 1: Transformations of rectangular axes, translation, rotation and their combinations; invariants.

Module 2: General equation of second degree and its various classifications (circle, ellipse, parabola, hyperbola, pair of straight lines etc); condition that the general equation of 2nd degree may represent two straight lines and some properties of the pair of straight lines.

Books Recommended

1. S.L. Loney: The Elements of Coordinate Geometry, McMillan.